An Investigation of Plume Rise

from

Titan IV Rocket Launches

THESIS

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Joseph Dwane Brands

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
In Partial Fulfillment of the
Requirements for the Degree of
Masters of Science in Engineering and Environmental Management

Member

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Chairman

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Joseph D. Brands, BS, MS
Captain, USAF

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ABSTRACT

Space launches at Cape Canaveral Air Station (CCAS) and Vandenberg Air Force Base (VAFB) produce exhaust ground clouds from the solid rocket boosters and liquid hypergolic fuels containing several toxic substances. In order to estimate the health effects that would be imposed upon the public by scheduled launches, range safety officials rely on the Rocket Exhaust Effluent Diffusion Model to predict ground level concentrations of these substances. A drawback to the REEDM is its underprediction of the initial ground clouds stabilization height. This underprediction causes an overprediction of the ground level toxic substance concentrations. This thesis focused on increasing the accuracy of the clouds stabilization height. Therefore, a model was developed incorporating conservation principles of volume, momentum, and buoyancy to predict stabilization height values. As part of the model a predictive function for the coefficient of entrainment was developed based on meteorological conditions. This rate of entrainment is a critical factor in accurately predicting the rise behavior of ground exhaust clouds.

The results of the model predicted the stabilization heights of grounds more accurately than the currently used REEDM model. The mean square difference between predicted and observed stabilization heights for five CCAS and two VAFB launches were determined. The mean square values for the model presented in this thesis and the REEDM, show a significant improvement of the model from the REEDM predictions.

I. INTRODUCTION

Overview

Space launches at Vandenberg Air Force Base (VAFB) and Cape Canaveral Air Station (CCAS) produce exhaust clouds from the solid rocket boosters and liquid hypergolic fuels. These fuels contain numerous toxic chemicals such as hydrogen chloride and hydrazine. The ignition of these fuels produce hazardous by-products such as hydrogen chloride gas (HC), carbon monoxide (CO), carbon dioxide (CO_2), and aluminum oxide particulates (Al_2O_3) which pose a risk to human health and welfare for individuals and the environment with which they come into contact. Due to this risk, safety personnel at the launch sites use a computer model which can predict the downwind ground level concentrations. The current model being used is the Rocket Effluent Exhaust Diffusion Model (REEDM). Based on the results of the REEDM the launch commander must decide whether the risk posed to human health is significant and whether the launch should be conducted as planned or whether it should be postponed until local meteorological conditions are favorable for launch.

One of the drawbacks with the REEDM model is that it uses conservative launch criteria which may predict ground level concentrations greater than what would actually occur. Therefore launches may be canceled or postponed unnecessarily at great expense to the Air Force. One of the major factors for the possible over prediction of ground level concentrations is the under prediction of the initial cloud stabilization height. The purpose of this paper is to develop a model which better predicts the stabilization height of the plume and therefore provide more accurate predictions of ground level

concentrations. This increased accuracy will provide launch commanders with more accurate information to base their decision of whether to go ahead with the launch or to delay it.

It is the intent of this thesis to develop a model which improves the accuracy of predicting the stabilization height of the launch cloud by incorporating better assumptions into a model based on the principles of conservation of momentum, volume, and buoyancy. The results of this model are compared to the predictions produced by the REEDM, a buoyancy formula prediction, and the observed plume stabilization heights for seven launches of the Titan IV rocket.

Practical Problem

Introduction

The problem facing the Air Force is significant as mission objectives are put at risk and large costs occur when launches are delayed or canceled. However, it is critical to realize that the safety of human health and welfare cannot be jeopardized by implementing a model which under predicts ground level concentrations. To fully understand the problem it is important to understand the various launch scenarios, which include a normal launch, conflagration failure, and deflagration failure scenarios.

Launch Scenarios

Launch commanders must consider three different launch scenarios for any given launch. The first is a normal launch scenario where the launch vehicle produces a large ground cloud in the initial moments of its launch. As the vehicle clears the ground then it

produces a line source as it continues to climb through the atmosphere. The ground cloud consisting of hot buoyant gases will rise up through the atmosphere until the density of the ground cloud equals that of the ambient atmosphere. The second is the conflagration failure scenario where the launch vehicle explodes scattering solid rocket motor fuel which continues to burn either on the ground or as it is falling to the ground. The third is deflagration failure scenario in which the rocket motor explodes in a large fireball consuming all the solid rocket motor fuel. The REEDM provides predictions for all three scenarios, however the efforts of this thesis are concentrated on the normal launch scenario.

Health Hazards

In a normal launch scenario various hazardous chemical are released which may pose a risk to populations surrounding the launch site. The hazardous chemicals of interest include hydrogen chloride gas (HCl), carbon monoxide (CO), carbon dioxide (CO_2), and aluminum oxide particulate (Al_2O_3). Currently, it is the risk of acute exposure to these chemicals which pose the highest risk to human health. In a normal launch scenario these chemicals are generated in concentrations sufficient to pose a risk to human health for the populations in close proximity. Hydrogen chloride gas is the most dangerous of the chemicals because a concentration of only 100 parts per million (ppm) (1: 126) is considered to be immediately dangerous to life and health (IDLH). It is therefore critical that accurate exposures can be predicted in order to ensure the safety of military personnel and populations close to the launch zone.

Academic Problem

The academic goal of this thesis is to increase the accuracy of the predicted ground cloud stabilization height, thereby allowing for a better prediction of downwind hazardous chemical concentrations. In order to accomplish this goal, a more detailed description of the manner in which cloud rise theory is applied in current prediction models is required.

Buoyancy Formula description

Morton, Taylor, and Turner (2: 19-21) studied the behavior of buoyant fluid rise by basing predictive models on simple conservation principles of volume, momentum, and density. In addition, they determined a simple buoyancy cloud rise formula, based on these conservation principles, which predicts a stabilization height of a buoyant cloud as a function of the heat contained within the cloud. This formula was derived based upon laboratory research and then applied to atmospheric conditions, and as such this formula is limited by the assumption that the atmosphere is completely stable and there is a positive gradient of potential temperature. Previous work accomplished by Sand (3: 1) used this buoyancy formula to predict stabilization heights of exhaust clouds from rocket launches.

REEDM Description

A second model used to predict cloud stabilization heights is the REEDM and is the model currently used by the U.S. Air Force. The REEDM computer program incorporates three main sections which include a meteorological model, a cloud rise

model, and a multi-layer diffusion model. Information concerning launch vehicle data, atmospheric meteorological data, and user supplied data are entered into the model to predict the downwind concentrations. These downwind concentration are given as toxic hazard corridors which indicate where downwind concentrations exceed permitted levels.

As stated, the REEDM program uses many subroutines in order to determine the toxic hazard corridors. It is the cloud rise model which is of interest in this thesis. The cloud rise algorithms used in REEDM are based on fundamental conservation equations and thermodynamic relationships. A critical parameter in these algorithms is the rate of entrainment of ambient air into the ground cloud. Entrainment of ambient air into the ground cloud reduces the buoyancy of the cloud as it rises through the atmosphere until there is no buoyancy force on the cloud. At this point the cloud will no longer rise and therefor has reached it's stabilization height. The REEDM currently employs a default value for this parameter which is the same for all launches. It is likely that the coefficient of entrainment is a function of meteorological conditions, therefore investigation of meteorological conditions and its effect on entrainment is of interest in this thesis.

Scope and Thesis Statement

The goal of this thesis is to reduce the accuracy in predicting the stabilization height of the initial ground cloud. In particular, this thesis will develop a model employing the basic principles of conservation for volume, momentum, and density. In addition observations will be made to gain insight into how the rate of entrainment is affected by

local meteorological conditions. The conditions of interest include the potential temperature gradient and wind shear in terms of velocity and direction. It is important to note that the REEDM code is used as a benchmark for comparisons in this thesis using the results of calculations performed by Sand (3: 35-40). The thrust of this thesis is to investigate how the aforementioned principles of conservation and effects of entrainment affect predicted ground cloud rise behavior and ultimately the ground cloud stabilization height for rocket launches.

Approach

The approach of this thesis involves several different efforts. First, the problem facing launch commanders at the two launch facilities is defined through a literature search and interviews with Air Force personnel. Second, a literature review of cloud rise theory is performed which develops an understanding of the behavior for cloud rise. Third, launch data of rocket launches is gathered including rocket exhaust characteristics, meteorological data taken during launches, observed ground cloud rise data, and predicted ground cloud rise behavior determined by the REEDM and the buoyancy formula. Fourth, a cloud rise program is developed based upon sound cloud rise theory which predicts cloud rise behavior. Fifth, the cloud rise program is executed and experiments on how the rate of entrainment could be better modeled are performed. The sixth and final step collects, analyzes, and discusses the results of the experiments.

Results

This thesis is organized into five chapters. The first chapter is the introduction which describes the problem addressed by this thesis. The second chapter is a literature review including the background theory this thesis uses as a foundation. The third chapter describes the methodology employed in developing the model. The fourth chapter presents the results of the efforts undertaken with this thesis. The fifth chapter is the conclusion and summarizes the findings and discusses the impact and insights garnered through the development of this thesis.

II. LITERATURE REVIEW

Introduction

To fully understand this thesis it is important to go into some detail concerning the background material on which it is based. Towards this end this chapter will chronicle the literature search conducted which is applicable to the focus of this thesis. The first area is that of information concerning space launches at VAFB and CCAS. Included in this will be a discussion of the exposure limit guidance which drives the need to accurately predict downwind toxic chemical concentrations, the current manner in which these exposure limits are characterized, and the meteorological data available for input into an air dispersion model. The second area of interest is a more detailed description of basic cloud rise theory. Finally, a brief presentation of the methodology for current air dispersions models is addressed. In particular, a description of the REEDM cloud rise computer sub-routines, the buoyancy formula, and the cloud rise model introduced in this thesis will be discussed.

Space Launch Issues

Exposure Limits

Space launch operations at VAFB and CCAS are seriously impacted by laws and regulations concerning exposure of humans to toxic chemicals. Human exposure limits applicable to range safety officials at VAFB and CCAS are a result of guidance put forth by the National Academy of Science (NAS), National Research Council's (NRC), Committee of Toxicology (COT), National Institute of Occupational Safety and Health

(NIOSH), and the U.S. Environmental Protection Agency (EPA) (4: 1). The COT guidance is primarily used for exposure to military personnel. As such assumptions are made concerning the population to include personnel of good health and women of child bearing age. The COT has also provided guidance to the DoD for exposure to the general public. The ceiling limits given are for short term, public emergency exposure guidance limits (SPEGLs). SPEGLs are determined based on acceptable exposures to peak concentrations for a single, unpredicted incident. Therefore the public can be exposed to the SPEGL for an isolated incident without risk to adverse human health affects. NIOSH has developed another type of exposure limit for toxic chemicals. The NIOSH levels are expressed as a dose immediately dangerous to life and health (IDLH) (5: WWWeb). The IDLH represents an maximum concentration to which an average person may be exposed to for 30 minutes without experiencing health effects which impairs an individuals ability to vacate the area. Finally, the EPA provides a third type of exposure limit as the level of concern (LOC) which is 1/10 the IDLH and represents a maximum level of concentration to which the public can be safely exposed for a one time occurrence. These limits are used by safety range officials to determine acceptable exposure levels for the three tier system detailed in the next section.

Toxic Hazard Corridor Estimations

The health risks due to toxic chemical exposures have been categorized into a three tier system, were each tier represents a toxic chemical concentration level (13: 32). The safety officials at the launch site then use the REEDM to determine predicted toxic chemical concentration levels for the area surrounding the launch site. This information is

used to determine the toxic hazard corridors or concentration isopleths. This three tier toxic corridor system is used by launch commanders at VAFB and CCAS to determine the potential risk of toxic exposure for local populations and as an aide into their launch decision.

Meteorological Data

Weather conditions at both launch sites are forecasted based upon meteorological conditions identified in pre-launch activities. These activities include data gather efforts of rawinsonde release, tower measurements, and sonic detection and ranging (SODAR) equipment. The meteorological measurements taken include wind speed, wind direction, air temperature, atmospheric pressure, and relative humidity. The measurements are taken at frequent intervals from ground level to approximately 3000 meters. Rawinsonde releases use weather balloons released into the atmosphere to gather data and occur at various times prior to launch with useable data available about 30 minutes after release (6: 83-94; 8: 99-112). SODAR equipment is used at VAFB along with meteorological measuring devices to determine wind speed, direction, and turbulence. Thirty minutes prior to launch at VAFB, SODAR measurements are taken to measure both vertical and horizontal wind turbulence. CCAS uses tower measurements to determine turbulence conditions prior to launch. It is important to note that the rawinsonde release method is the only meteorological measurement method employed at both sites and for all launches. This is important as it provides the only consistent data gathered and available for analysis.

VAFB typically experiences a mixing depth of 150 to 200 meters up into the atmosphere. A mixing depth represents a height into the atmosphere where a stable region of air is encountered. Motion across and through this layer is impeded by the stable nature of the atmosphere at this elevation. At VAFB nocturnal inversions are also common, due to low wind levels, and produce mixing depths of less than or equal to 100 meters. These nocturnal inversions nearly eliminate any vertical turbulence in the atmosphere (14: 39). It is also common for VAFB to experience inversion layers reaching an elevation of 1.5 kilometers. Mixing depths of this height make meteorological data in this region critical to ensure that all inversion layers are detected and accounted for in modeling scenarios. This is critical for gaseous diffusion modeling as inversion layers act as impenetrable barriers to vertical mixing. However, due to the high initial heat of rocket exhaust ground clouds these clouds contain enough buoyancy to rise through low inversion layers. An additional meteorological condition of interest at VAFB is an atmospheric transition zone caused by land-sea breeze effects which can cause considerable wind speed and direction shears. These shears can cause significant vertical and horizontal turbulence.

Basic Cloud Rise Theory

Introduction

To understand the various cloud rise models used in this thesis it is important to have a basic understanding of cloud rise theory. The classical meteorological treatment of parcel motion needs to be addressed, as it lays the foundation for cloud rise. Specifically the of

the laws of thermodynamics, the relationship between potential temperature and entropy, and the hydrostatic atmosphere assumption are addressed.

Thermodynamic Relationships

The first law of thermodynamics is an expression of the conservation of energy in its various forms and can be written as a rate equation (15: 73-77)

$$\overset{\circ}{Q} = \frac{dU}{dt} + \frac{d(KE)}{dt} + \frac{d(PE)}{dt} + \overset{\circ}{W} \tag{2-1}$$

where,

$\overset{\circ}{Q}$	=	rate of heat transfer to the system
$\frac{dU}{dt}$	=	rate of change of internal energy of the system
$\frac{d(KE)}{dt}$	=	rate of change of kinetic energy of the system
$\frac{d(PE)}{dt}$	=	rate of change of potential energy of the system
$\overset{\circ}{W}$	=	rate of work done by the system

The system is a parcel of air of sufficient size to be of continuous density, temperature, and pressure, yet small enough to be homogenous with regard to these physical properties. The first law is used to evaluate exchanges between the parcels velocity, height, and heat.

The second law of thermodynamics defines the entropy of the system and relates the direction in which the process will occur. Entropy can be thought of as a measure of the

randomness of a system on a microscopic level. Therefore as a systems temperature increases the molecular motion increases thereby increasing its entropy. The second law can be represented with the following inequality:

$$dS \geq \frac{\delta Q}{T} \tag{2-2}$$

where,

dS	=	change in entropy
δQ	=	change in heat content
T	=	temperature

The equality condition holds only for idealized processes that are reversible. The second law of thermodynamics can also be defined using the *Clausius statement* stated simply that it is impossible for any system to operate in such a way that the sole result would be an energy transfer by heat from a cooler to a hotter body (16: 167-168). This leads to an important principle:

$$dS_{system} + dS_{surroundings} \geq 0 \tag{2-3}$$

This inequality means that the only processes which can occur in the atmosphere is one which maintains or increases entropy. Entropy can be expressed as a function of internal energy (u) and specific volume (ν). The total derivative of the function s(u,ν) is as follows.

$$ds = \left(\frac{\partial s}{\partial u}\right)_{\nu} du + \left(\frac{\partial s}{\partial \nu}\right)_{\nu} d\nu \tag{2-4}$$

Using the thermodynamic definitions of temperature (T) and pressure (P) with equation (2-4) results in the Gibbs' equation.

$$ds = \frac{1}{T}du + \frac{P}{T}dv \qquad (2\text{-}5)$$

The Gibbs equation is useful in that it provides a way in which to compute entropy from measurable meteorological conditions namely temperature and pressure.

For the study of cloud rise it is also useful to use the definition of specific heat C_p and assume that the atmosphere obeys the ideal gas law. The ideal gas law and C_p are defined as,

$$Pv = RT \qquad (2\text{-}6)$$

where,

R = gas constant for air (287 $JKg^{-1}\,K^{-1}$)

$$C_p = \frac{dh}{dT} \qquad (2\text{-}7)$$

where,

$$dh = du + Pdv + vdP$$

Incorporating equations (2-6) and (2-7) the entropy change may be written as:

$$ds = \frac{dh}{T} - \frac{Pdv}{T} - \frac{vdP}{T} + \frac{Pdv}{T} = c_p\frac{dT}{T} - R\frac{dP}{P} \qquad (2\text{-}8)$$

Integrating equation (2-8) gives,

$$s - s_o = C_p\left(\ln\frac{T}{To}\right) - R\left(\ln\frac{P}{P_o}\right) = C_p\left[\ln\left(\frac{T}{T_o}\right)\left(\frac{P_o}{P}\right)^{\frac{R}{Cp}}\right] \qquad (2\text{-}9)$$

Solving for the final temperature (T) gives:

$$T = T_o\left(\frac{P}{P_o}\right)^{\frac{R}{Cp}\exp\left[\frac{s-so}{Cp}\right]} \qquad (2\text{-}10)$$

Finally, if it is assumed that the parcel motion is reversible so entropy remains constant and adiabatic so no heat is transferred between the parcel and the atmosphere then the process is isentropic or $(s - s_o = 0)$. With this assumption the parcel temperature can be calculated using the following equation.

$$T = T_o\left(\frac{P}{P_o}\right)^{\frac{R}{Cp}} \qquad (2\text{-}11)$$

Relationship Between Potential Temperature and Entropy

Meteorologists use the concept of potential temperature (θ) to define entropy changes. The potential temperature effectively normalizes the temperature with respect to pressure. To define entropy changes with potential temperature the following equation applies,

$$s - s_o = C_p\left[\ln\left(\frac{\theta}{\theta_o}\right)\right] \qquad (2\text{-}12)$$

Comparing this with equation (2-11) which assumes the parcel behaves as an ideal gas the potential temperature can be expressed as,

$$\frac{\theta}{\theta_o} = \frac{T}{T_o}\left(\frac{P_o}{P}\right)^{\frac{R}{Cp}} \qquad (2\text{-}13)$$

The convention is to assign $P_o = 1000$ millibars and $\theta_o = T_o$ yielding,

$$\theta = T\left(\frac{1000}{P}\right)^{\frac{R}{Cp}} \qquad (2\text{-}14)$$

Finally, the second law can be expressed as a function of potential temperature.

$$\frac{d\theta}{dt} = \theta_o e^{\left(\frac{s-s_o}{C_p}\right)} \frac{1}{C_p}\frac{ds}{dt} \geq 0 \qquad (2\text{-}15)$$

At this point if a relationship between pressure and altitude can be derived, the potential temperature as a function of height can be defined. This is known as the potential temperature lapse rate.

Potential Temperature Lapse Rate

Cloud rise theory makes the assumption that the atmosphere can be treated as though it were at rest for the purpose of relating pressure to altitude. For an atmosphere at rest, the hydrostatic pressure at height (z) becomes the weight of the air column above the height (z). This can be stated as,

$$p = \int_{\tau=z}^{\tau=\infty} \rho g d\tau \qquad (2\text{-}16)$$

This pressure relationship can be written as the total derivative

$$dp = \frac{\partial P}{\partial t}dt + \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz \qquad (2\text{-}17)$$

This total derivative can be simplified employing the hydrostatic pressure relationship which presumes that $\partial P / \partial t$, $\partial P / \partial x$ and, $\partial P / \partial y$ equal zero.

The potential temperature lapse rate can be derived by combining the hydrostatic pressure relationship with the definition of potential temperature. In order to do this the potential temperature equation is first logarithmically differentiated to give the form,

$$\frac{1}{\theta}\frac{\partial\theta}{\partial z}=\frac{1}{T}\frac{\partial T}{\partial z}-\frac{R}{C_pP}\frac{\partial P}{\partial z} \tag{2-18}$$

substituting,

$$-g\rho=-\frac{gP}{RT}$$

for,

$$-g\rho=\frac{\partial P}{\partial z}$$

results in,

$$\frac{\partial\theta}{\partial z}=\frac{\theta}{T}\left[\frac{\partial T}{\partial z}+\frac{g}{C_p}\right]=\frac{\theta}{T}\left[\gamma_d-\gamma\right] \tag{2-19}$$

where,

$$\frac{\partial\theta}{\partial z} \quad = \quad \text{potential temperature lapse rate}$$

$$\gamma_d=\frac{g}{C_p}\approx\frac{10°\mathrm{K}}{\mathrm{km}} \quad = \quad \text{dry adiabatic lapse rate}$$

$$\gamma=-\frac{\partial T}{\partial z} \quad = \quad \text{lapse rate}$$

With the above equations the potential temperature lapse rate becomes a useful measure of atmospheric stability and will be explored in the next section.

A Linear Model of Vertical Parcel Motion

In the most basic sense, in absence of other external forces, vertical parcel motion occurs when the density of parcel is less than that of the surrounding ambient atmosphere. Under these conditions a buoyancy force acts on the parcel. Employing Archimede's principle, this buoyancy force is directly proportional to the force of gravity times the

difference in mass of the parcel (M_p) and that of the air displaced by the parcel (M) (22: 475).

$$F_b = g(M - M_p) \tag{2-20}$$

Employing Newton's second law of motion, the parcel acceleration can be determined.

$$F_b = M_p a = M_p \frac{d^2 z}{dt^2} \tag{2-21}$$

By equating these two equation and dividing by the parcel volume, the vertical acceleration can be expressed as a function of the density differences.

$$\frac{d^2 z}{dt^2} = g\left(\frac{\rho - \rho_p}{\rho_p}\right) \tag{2-22}$$

At this point in vertical parcel motion theory two assumptions are made.

1. The parcel is at equilibrium in terms of pressure at any give height (z).
2. The parcel motion occurs without any heat transfer with the atmosphere and the process is isentropic and therefore has constant potential temperature.

Employing these assumptions, the ideal gas law (2-6) and potential temperature equation (2-13) yield the following.

$$\frac{\rho_p}{\rho} = \frac{T}{T_p} = \frac{\theta}{\theta_p} \tag{2-23}$$

Where the subscripted values indicate the properties of the parcel and unsubscripted values represent the properties of the ambient atmosphere. With equation (2-31) the vertical acceleration of a parcel can be expressed as the following.

$$\frac{d^2 z}{dt^2} = g\left(\frac{\theta_p - \theta}{\theta}\right) \tag{2-24}$$

From the above equation it can be seen that if the potential temperature of the parcel is greater than the surrounding air, the parcel will have an upward acceleration and alternately if the potential temperature of the parcel is less than the surrounding air the parcel will have a downward acceleration. This information can be used to classify stability classes pertaining to vertical motion. The following inequality equations are typically used in meteorology to establish stability classes.

$$\text{stable:} \qquad \frac{\partial \theta}{\partial z} > 0$$

$$\text{neutral:} \qquad \frac{\partial \theta}{\partial z} = 0$$

$$\text{unstable:} \qquad \frac{\partial \theta}{\partial z} < 0$$

For this analysis it is assumed the parcel is initially at equilibrium with the surrounding atmosphere at the point prior to displacement. To illustrate this, if the atmosphere is stable and the parcel is displaced upward, then the parcel will cool faster then the surrounding air and therefore will have a higher density then the surrounding air causing it to sink down to the height where there is no difference in density between the atmosphere and the parcel. In this way, a stable atmosphere acts as a restoring force, thereby dampening vertical turbulence. In a neutral atmosphere the parcel will cool at the same rate as the atmosphere and will eventual stop its upward movement due to frictional forces. Finally, in an unstable atmosphere the parcel will cool slower than the atmosphere, resulting in the parcel having a lower density than the surrounding air. This results in the parcel experiencing a buoyant force which will accelerate the parcel upward.

Stability Parameter

The following derivation describes the stability parameter based on the potential temperature profile of the atmosphere. To begin, the potential temperature of the atmosphere can be approximated using a Taylor series expansion. For small differences in height we can drop the higher ordered terms which results in the following equation.

$$\theta - \theta_o = \frac{\partial \theta}{\partial z}(z - z_o) \tag{2-25}$$

The acceleration of a parcel with a small change in height can then be approximated as:

$$\frac{d^2(z - z_o)}{dt^2} = -g\left(\frac{\theta - \theta_o}{\theta}\right) \cong -\frac{g}{\theta}\left(\frac{d\theta}{dz}\right)(z - z_o) \tag{2-26}$$

Substituting η for $z - z_o$, the linear homogenous second order differential equation results:

$$\frac{d^2\eta}{dt^2} + \frac{g}{\theta}\left(\frac{\partial \theta}{\partial z}\right)_o \eta = 0 \tag{2-27}$$

This has a solution of the form $\eta = Ae^{\lambda t}$, with a characteristic equation of

$$\lambda^2 + \frac{g}{\theta}\left(\frac{\partial \theta}{\partial z}\right)_o = 0 \tag{2-28}$$

Solving for λ gives:

$$\lambda = \pm\sqrt{\frac{g}{\theta}\left(\frac{\partial \theta}{\partial z}\right)_o} = \pm w_{BV} \quad \text{for} \quad \frac{\partial \theta}{\partial z} < 0 \tag{2-29}$$

$$\lambda = \pm i\sqrt{\frac{g}{\theta}\left(\frac{\partial \theta}{\partial z}\right)_o} = \pm i w_{BV} \quad \text{for} \quad \frac{\partial \theta}{\partial z} > 0 \tag{2-30}$$

The term w_{BV} has the units of (1/sec) and is known as the Brunt-Väisälä frequency. The Brunt-Väisälä frequency captures the physical dynamic that the parcel will oscillate about the height where there is no density difference between the parcel and the atmosphere until frictional forces cause the parcel to remain at the height of no density difference. For the case of a stable atmosphere $\left(\partial\theta/\partial z > 0\right)$ the solution for η is,

$$\eta = Ae^{iw_{BV}t} + Be^{-iw_{BV}t} \tag{2-31}$$

The coefficients A and B are found using the initial conditions at t = 0, namely where,

$$\frac{d\eta}{dt} = W_o, \text{ and } \eta_o = 0, \text{ where } W_o = \text{initial displacement velocity}$$

from which

$$A = \frac{W_o}{2iW_{BV}}, \quad B = \frac{W_o}{2iW_{BV}}. \tag{2-32}$$

Therefore,

$$\eta = \frac{W_o}{2iW_{BV}}\left[e^{iw_{BV}t} - e^{-iw_{BV}t}\right]. \tag{2-33}$$

If we now apply Eulers identity we get,

$$\eta = \frac{W_o}{W_{BV}}\sin\left(W_{BV}t\right). \tag{2-34}$$

If the initial height is assigned to be zero then this equation forms a relationship between height and the potential temperature lapse rate. This can then be used to predict the height a buoyant air parcel will rise. It is also important to note that the height z is periodic with respect to time and will reach a maximum value when,

$$W_{BV}t = \left(\sqrt{\frac{g}{\theta}\frac{\partial\theta}{\partial z}}\right)_o t = \frac{\pi}{2} \tag{2-35}$$

For the case of an unstable atmosphere the equation for η is,

$$\eta = \frac{W_o}{2\left|\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right|_o^{1/2}}\left[e^{\left|\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right|_o^{1/2} t} - e^{-\left|\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right|_o^{1/2} t}\right] \tag{2-36}$$

Inspection of this equation illustrates that the first term will grow exponentially while the second goes to zero and confirms the instability of the parcel when the potential lapse rate is negative.

The preceding analysis represents a relationship predicting the height a parcel will achieve given an initial displacement velocity and the potential temperature lapse rate. A similar type of analysis can be accomplished for a parcel with a given initial buoyancy due to the parcel having a different potential temperature than the surrounding air. This is the case which more accurately represents conditions during a rocket launch, as the rocket exhaust has a much higher temperature than the surrounding atmosphere. In order to develop this relationship the assumptions of isentropic displacement with no entrainment of surrounding air into the parcel and the hydrostatic atmosphere are required. With these assumptions,

$$\frac{d^2\eta}{dt^2} + \frac{g}{\theta}\left(\frac{\partial\theta}{\partial z}\right)_o \eta = g\left(\frac{\theta_p - \theta_o}{\theta_o}\right) \tag{2-37}$$

where,

$$g\left(\frac{\theta_p - \theta_o}{\theta_o}\right) \quad = \quad \text{initial acceleration at } t = 0$$

$$\theta_p \quad = \quad \text{initial parcel potential temperature}$$

$$\theta_o \quad = \quad \text{initial ambient potential temperature}$$

This is a non-homogeneous differential equation with constant coefficients. The solution to this equation for a stable atmosphere is,

$$\eta = \frac{g}{w_{BV}^2}\left(\frac{\theta_p - \theta_o}{\theta_o}\right)\left(1 - \cos w_{BV} t + i \sin w_{BV} t\right) \tag{2-38}$$

The real part of this equation defines a cyclic function, which predicts the height a buoyant parcel will reach given an initial buoyancy and given potential lapse rate. The maximum height occurs when $\cos w_{BV} = -1$.

The preceding discussion of the linear model for parcel motion has introduced the concepts of buoyancy, acceleration, and potential temperature. These ideas will be used in the next section form the basis of the cloud rise algorithms used in the REEDM, as well as the derivations for the buoyancy formula and the differential equation model presented in this thesis.

Meteorological Parameter Effects on Entrainment and Cloud Rise

Entrainment is the phenomena in which ambient air is mixed with gases contained in a cloud. In the case of a rocket exhaust cloud entrainment mixes cooler ambient air into the exhaust cloud thereby reducing its buoyancy. Several different meteorological

parameters have an effect on this rate of entrainment. Some of these parameters serve to reduce the entrainment thereby holding the cloud together and retaining its buoyancy while others serve the opposite function. Some of the critical meteorological parameters of interest for this purpose are the potential temperature lapse rate, wind speed, wind direction, dew point temperature, and relative humidity. This section will discuss the qualitative affects these parameters have on the rate of entrainment.

The potential temperature lapse rate introduced earlier in this chapter is a common parameter used to classify the stability of the atmosphere. As mentioned earlier unstable atmospheric conditions occur when the potential temperature lapse rate is negative. In an unstable atmosphere a parcel of air set in vertical motion by some sort of turbulence will tend to continue its motion (18: 2). This has the effect of increasing the amount of turbulent motion in the atmosphere which thereby increases the amount of entrainment. If the atmosphere is stable then the opposite is true. A parcel of air set in vertical motion by some kind of turbulence will tend to return to the elevation from where it was displaced. For this reason it is likely that the stability of the atmosphere has a strong affect on the rate of entrainment.

Wind speed also seems to exhibit an affect on atmospheric turbulence and thereby the rate of entrainment. Wind persists in the atmosphere due to relative air pressure differences. Wind is caused by an area of relatively high pressure moving to an area of relatively low pressure. Due to ground effects in the boundary layer the nature of wind is especially chaotic. This causes the wind to change speed and direction in a relatively

small amount of space. This then causes shearing forces in the atmosphere which causes the atmosphere to become turbulent (19: 66-69). If a buoyant cloud is rising through this turbulence, as is the case for a rocket exhaust cloud, the rate of entrainment of ambient air into the cloud will increase. The stronger the winds the more profound the effect on the rate of entrainment. In this sense, it is expected that strong winds increase the amount of entrainment experienced by a rocket exhaust cloud rising through the atmosphere.

The dew point is the temperature to which moist air must be cooled at constant pressure and moisture content for it for it to reach saturation (20: 19). This means that water vapor condenses when the temperature of the parcel reaches the dew point. If the temperature of the air is above the dew point evaporation will occur. The evaporation process takes energy, in the form of heat, from its surroundings. Therefore, the greater the difference between the dew point and the temperature of the air the more evaporation occurs. In the case of a rocket exhaust cloud from a Titan IV rocket the temperature of the exhaust cloud is approximately the same from launch to launch. As a result, the lower the ambient dew point the more evaporation will occur. This evaporation uses the heat contained in the cloud reducing its own buoyancy. This affects the cloud such that it retards the vertical motion and consequently reduces the clouds ability to rise through the atmosphere. The relative humidity has the same effect as the dew point. Relative humidity represents the amount of water vapor contained in the atmosphere. At 100% relative humidity the atmosphere is saturated and evaporation cannot occur; therefore, the lower the relative humidity the more evaporation will take place with the same effect on

the rise of the cloud. In summary, the lower the dew point and relative humidity the quicker the buoyancy force will dissipate and the less energy the cloud has to be propelled upward.

Discussion of Cloud Stabilization Height Theory

Discussion of Conservation Equations

Morton, Taylor, and Turner presented a method to determine stabilization heights in the case where a lighter fluid is discharged into a heavier, stably stratified fluid with which it can mix. This scenario can also be simulated through a sudden release of heat into the fluid thereby causing a cloud of heated fluid to rise through the stably stratified environment. Morton et al. used equations representing conservation of volume, momentum, and heat to study vertical convection in an environment of an incompressible fluid. These conservation principles were then used to present a series of differential equations for both continuous and instantaneous sources. The system of equations presented for an instantaneous source are as follows.

$$\text{(i)}\quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \alpha k u, \qquad \text{(conservation of volume)} \qquad (2\text{-}29)$$

$$\text{(ii)}\quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho k u\right) = \frac{4}{3}\pi r^3 k g\left(\rho - \rho_o\right), \qquad \text{(conservation of momentum)}$$

$$\text{(iii)}\quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3 k g \frac{\rho_o - \rho}{\rho_1}\right) = -\frac{4}{3}\pi r^3 G u, \qquad \text{(conservation of heat)}$$

$$\text{(iv)}\quad \frac{dx}{dt} = u \qquad \text{(velocity)}$$

Where,

r = cloud radius

α = coefficient of entrainment

k = unit vector conversion factor equal to 1

u = vertical velocity

g = acceleration due to gravity

ρ = cloud density

ρ_o = atmospheric density

ρ_1 = reference density

$$G = -\frac{g}{\rho_1}\frac{d\rho_o}{dx}$$

These equations are based on two main assumptions.

i. The rate of entrainment at the edge of the plume or cloud is proportional to some characteristic velocity at that height.

ii. The largest local variations of density in the field of motion are small in comparison with some chosen reference density.

These conservation equations, when solved for provide a method to predict stabilization heights of exhaust clouds generated by rocket launches.

Simple Cloud Rise Formulas

The buoyancy of a cloud is a significant factor in determining the stabilization height of clouds. Using this fact, a lot of research has been accomplished attempting to develop a simple formula to predict stabilization based on the buoyancy of a cloud. Methods for

doing this have been developed over time resulting in numerous solutions, too many to be listed here in an exhaustive manner (21: 59). The problem is that many of these methods are based on empirical findings and limiting assumptions, therefore they do not agree with each other or with observations except on an approximate basis. However, these simple formula can still be a useful comparison for new methods. One in particular, introduced by Morton et al., uses the heat released into a cloud to predict a final stabilization height.

$$H = 1.87Q^{1/4} \tag{2-40}$$

Where H is the final height in meters and Q is the rate of heat released in Joules. This can be applied to rocket launches as the heat generated in a launch is a known quantity.

The REEDM Cloud Rise Formula for Instantaneous Sources

The current model used by the U.S. Air Force is the REEDM. The developers of the REEDM used a linear model approach which is similar to the linear model of vertical parcel motion described in the previous section. The REEDM also employs other cloud rise algorithms to model launch scenarios including explosions and prolonged burning of fuels. These latter scenarios are not the focus of this thesis and therefore those algorithms will not be discussed. The REEDM uses the algorithms for cloud rise in an iterative manner which allow for air entrainment to be included and the subsequent reduction of buoyancy as the cloud rises.

Following the work performed by Morton et al. (2: 1) the instantaneous cloud rise equation in the REEDM employs Newton's second law of motion for the relationship between the buoyancy force and the vertical momentum.

$$F_B = \frac{d}{dt}\left(M_c w\right) \qquad (2\text{-}41)$$

where,

$$w = \frac{dz}{dt} \qquad \text{(vertical velocity)}$$

In equation (2-39) the buoyancy force is,

$$F_B = g\left(M - M_c\right) \qquad (2\text{-}42)$$

In equation (2-40) the subscript refers to the mass of the cloud and the unsubscripted variable refers to the mass of the displaced ambient air. If we use the relationship between mass, density, and volume and if the cloud is assumed spherical we can determine the buoyancy force for a spherical cloud.

$$F_B = g\left(M - M_c\right) \qquad (2\text{-}43a)$$

$$F_B = Vg\left(\rho - \rho_c\right) \qquad (2\text{-}43b)$$

$$F_B = \frac{4}{3}\pi r^3\left(\rho - \rho_c\right) \qquad (2\text{-}43b)$$

Using equations (2-39) and (2-41b) results an equation for momentum (m):

$$m = \frac{4}{3}\pi r^3 \rho_c w \qquad (2\text{-}44)$$

Dividing equation (2-42) by ($4/3\pi\rho_c$) gives the basic REEDM equation.

$$br^3 = \frac{1}{\rho_c}\frac{d}{dt}\left(\rho_c wr^3\right) \tag{2-45}$$

where

$$b = g\left(\frac{\rho - \rho_c}{\rho_c}\right) \approx g\left(\frac{\rho - \rho_c}{\rho}\right)$$

r = cloud radius

The buoyancy force acting on a cloud that rises with constant potential temperature decreases as a function of the atmospheric lapse rate ($\gamma = d\theta/dz$) in the following manner.

$$\frac{dF_B}{dz} = \frac{d}{dz}\left[\frac{M_c g}{\theta}\left(\theta_p - \theta\right)\right] \cong -M_c \frac{g}{\theta}\frac{d\theta}{dz} \tag{2-46}$$

The above equation assumes the cloud rises with constant mass and in a isentropic manner. The REEDM authors then apply the chain rule to obtain the following equation.

$$\frac{dF_B}{dt} = \frac{dF_B}{dz}\frac{dz}{dt} = -\frac{4}{3}\pi r^3 \rho_c \frac{g}{\theta}\frac{d\theta}{dz}w = -\frac{4}{3}\pi r^3 \rho_c sw \tag{2-47}$$

Where s is the stability parameter used by the REEDM and equals the following.

$$s = \frac{g}{\theta}\frac{d\theta}{dz}$$

The rate of change of the buoyancy force is equated to the second derivative of momentum to yield.

$$\frac{d}{dt}\left(br^3\right) = \frac{d^2}{dt^2}\left(wr^3\right) = -swr^3 \tag{2-48}$$

where $\frac{4}{3}\pi\rho_c$ has been divided out of the equation. The middle term can be transformed again by using the chain rule twice resulting in the following form of the equation.

$$\frac{d^2}{dt^2}\left(wr^3\right) = u^2\left[\frac{d^2}{dx^2}\left(wr^3\right)\right] \tag{2-49}$$

where dx/dt = u is the mean steady state horizontal wind speed. The final form of the cloud rise equation is,

$$u^{-2}\frac{d^2\left(wr^3\right)}{dx^2} = -s\left(wr^3\right) \tag{2-50}$$

This equation is a second order differential equation with the following solution, if s is assumed to be constant.

$$wr^3 = F_m \cos\left(\frac{\sqrt{s}x}{u}\right) + \frac{F_I}{\sqrt{s}}\sin\left(\frac{\sqrt{s}x}{u}\right) \tag{2-51}$$

Where,

$F_m = w_o r_o^3$ initial momentum (constant)

$F_I = br^3 \approx \dfrac{3gQ_I}{4C_p\pi\rho T}$ initial buoyancy (constant)

$Q_I =$ initial heat release to create the buoyant cloud

Entrainment is the physical phenomena whereby atmospheric air surrounding the cloud is mixed into the cloud through turbulence. Specifically, turbulent eddies coming in contact with the edge of the cloud will mix ambient air into the cloud. As the atmospheric air is cooler and denser than that of the cloud, the buoyancy of the cloud decreases as a function of the rate of entrainment. The REEDM code uses work accomplished by Morton et al., to model the growth of the cloud in a linear form and as a function of height.

$$r = r_o + \alpha z \tag{2-52}$$

where,

$r =$ the cloud radius

$r_o =$ the initial cloud radius

$\alpha =$ an empirical entrainment coefficient

The value used in the REEDM was arbitrarily decided to be an $\alpha = 0.64$. Morton, Taylor, and Turner (2: 19) performed some laboratory work to determine an appropriate value for the coefficient of entrainment. Their study concluded that for a stable stratified environment an appropriate value for this coefficient should be an α varying from 0.27 to 0.34. It is clear however, that any atmosphere outside of laboratory conditions will not be stable and stratified completely therefore a value greater than the 0.27 is appropriate.

To determine the equation used by the REEDM, equation (2-50) is substituted into equation (2-49) and then is integrated. This results in an equation which predicts the height an instantaneous ground cloud will rise.

$$z = \left[\frac{4F_m}{\alpha^3\sqrt{s}}\sin\left(t\sqrt{s}\right) + \frac{4F_I}{\alpha^3 s}\left(1 - \cos\left(t\sqrt{s}\right)\right) + \left(\frac{r_o}{\alpha}\right)^4\right]^{1/4} + \frac{r_o}{\alpha} \qquad \text{(2-53)}$$

This is the form of the cloud rise equation employed by the REEDM which is similar to the linear model described earlier while being modified by the air entrainment assumption. It is important to reiterate the assumptions for which this equation is appropriate.

1. The mass of the parcel is constant.
2. The cloud rise process is adiabatic and reversible (i.e. isentropic).

3. No water vapor phase change.

4. The stability factor $(g / \theta) d\theta / dz$ is constant over the cloud trajectory.

5. Horizontal wind speed is constant over time and height of the cloud trajectory.

6. Stable atmosphere ($s > 0$).

7. Entrainment coefficient is constant over the cloud trajectory.

The above assumptions applied to the cloud rise equation (2-51) seem to be restrictive when applied to real world conditions. In an attempt to minimize this affect the REEDM algorithms employs an iterative scheme to move the buoyant plume through its rise. The algorithm takes vertical steps whereby conditions requiring constant values are assigned the average values associated with that layer. In this way the REEDM is better able to approximate cloud rise behavior and the final stabilization height of a rocket exhaust cloud.

III. METHODOLOGY

Introduction

The approach, used in this thesis, for investigating the instantaneous ground cloud rise is based on a system of differential equations derived by Morton et al. from basic principles of the conservation of volume, momentum, and density. A numerical algorithm has been implemented in Mathcad™ to solve these equations. This algorithm is called the model in the remainder of this thesis. The model is then enhanced so it can predict stabilization heights based only on information known prior to launch. In order for the model to have this capability values must be known for its three input parameters. These parameters include an initial ground cloud radius, the density change in the ambient atmosphere as a function of height, and the coefficient of entrainment. This chapter will discuss the methods used to determine values for each of these parameters. In addition, this chapter will also detail the launches used as test cases for its development.

Selection of Test Cases

Many rocket launches have occurred at VAFB and CCAS over the past years, however not all have cloud rise observation measurements. This observation data is critical in testing and validating any cloud rise model because of its utility in providing data required to develop a model. The Titan IV rocket program is therefore of prime interest as seven separate launches have observed cloud rise data measurements along with the corresponding meteorological measurements. This data can be found in Appendices A

and B. Five launches occurred at CCAS and are identified as K-2, K-16, K-19, K-21, and K-23, while two occurred at VAFB and are identified as K-15 and K-22. The observations were made using both visual and infra-red imagery techniques or a combination of the two. These data have then been used to determine curve fit equations for cloud rise as a function of time. This equation therefore can be used to plot a profile of the height of the cloud as a function of time and can be used to compare results from cloud rise prediction models. In addition to the curve fit equation, the reports and memorandums provide summary data for the ground cloud stabilization heights for the bottom, centerline, and top of the ground cloud (7: 16-18; 8: 17-24; 9: 16-18; 11: 11-13; 12: 15-18) and are presented in Table 3-1.

Mission	K-2	K-15	K-16	K-19	K-21	K-22	K-23
B_{ottom} (m) Visible	1370	383	219	1080	886	164	1152
B_{ottom} (m) I/R	1370	N/A	N/A	1394	886	164	1152
H_{center} (m) Visible	1871	658	1023	1774	1375	702	1640
H_{center} (m) I/T	1864	N/A	N/A	1913	1375	702	1640
H_{top} (m) Visible	2635	1172	1877	2482	2120	1264	2211
H_{top} (m) I/R	2635	N/A	N/A	2433	2120	1264	2211
Date	7/2/96	12/5/95	4/24/96	7/10/95	11/6/95	5/12/96	5/14/95

Table 3-1: Measured Titan IV Cloud Stabilization Heights (meters)

The five launches at CCAS are used as the primary focus for development of the model. This is done because they provide enough cases to develop a model upon. The VAFB launches are then used as independent test case for the predictive model. This will help to validate to some extent the robustness of the model in predicting stabilization heights for launches at locations other than CCAS.

Basic Differential Equation Model

Development of Differential Equations

Morton, Taylor, and Turner developed a system of differential equations which are used to predict the stabilization height of an instantaneous buoyant cloud (2: 15-19). This system of equations are based on fundamental principles of conservation. These fundamental principles include the conservation of volume, momentum, and density deficiency. The fourth fundamental quantity used in the system is that of vertical velocity. The vertical motion of the cloud is driven by a buoyancy force which is due to a density difference between gas inside the cloud and that of the ambient atmosphere. In the process of rocket launches, a ground cloud of heated exhaust is quickly formed before the rocket achieves sufficient altitude to no longer contribute exhaust gases to the ground cloud. An important assumption is that the time in which the ground cloud is formed is quick enough to assume it is created instantaneously.

The instantaneous ground cloud begins it vertical motion and rises into the atmosphere. As it does so the turbulent nature of the ground cloud and the atmosphere cause the gases of the cloud and the surrounding air to mix. Ambient air is denser than that of the cloud

gases, therefore as entrainment occurs the density of the cloud will increase. This mixing of ambient air into the cloud is termed entrainment and is one of the braking forces which slows vertical motion eventually causing the cloud to reach a stabilization height. The other force slowing the vertical motion of the cloud is due to the decreasing density of the atmosphere as a function of height. In a stable atmosphere, pressure decreases with height thereby causing the density of the air to also decrease. This decrease in atmospheric density coupled with the increase in cloud density due to entrainment drives the density difference to zero, at which point the cloud has reached a stable altitude.

The conservation equations employed by Morton et al. (2: 15-16) are given as equation (2-53) in Chapter 2 and are provided here again for convenience.

$$\text{(i)} \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \alpha k u, \qquad \text{(conservation of volume)} \qquad (2\text{-}39)$$

$$\text{(ii)} \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho k u\right) = \frac{4}{3}\pi r^3 k g(\rho - \rho_o), \qquad \text{(conservation of momentum)}$$

$$\text{(iii)} \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3 k g \frac{\rho_o - \rho}{\rho_1}\right) = -\frac{4}{3}\pi r^3 G u, \qquad \text{(conservation of heat)}$$

$$\text{(iv)} \quad \frac{dx}{dt} = u \qquad \text{(velocity)}$$

Equation (2-53) can be simplified to the following system of differential equations.

$$\text{(i)} \quad \frac{d}{dt}\left(r^3\right) = 3r^2 \alpha u, \qquad (3\text{-}1)$$

$$\text{(ii)}\ \frac{d}{dt}\left(r^3\rho u\right) = r^3 g\left(\rho - \rho_o\right)$$

$$\text{(iii)}\ \frac{d}{dt}\left(r^3 g\frac{\rho_o - \rho}{\rho_1}\right) = -r^3 Gu,$$

$$\text{(iv)}\ \frac{dx}{dt} = u$$

where k is a unit vector equal to one for this case and,

$$G = -\frac{g}{\rho_1}\frac{d\rho_o}{dx}\ .$$

If,

$$v_1 = r^3$$

$$v_2 = r^3\rho u$$

$$v_3 = r^3 g\frac{\rho_o - \rho}{\rho_1}$$

$$v_4 = x$$

and substitute into equation (3-1) the following results:

$$\text{(i)}\ \frac{d}{dt}\left(v_1\right) = 3r^2\alpha u = 3r^2\alpha u \qquad \text{(3-2)}$$

$$\text{(ii)}\ \frac{d}{dt}\left(v_2\right) = v_1\left(\rho_o - \rho\right) = \rho_1 v_3$$

$$\text{(iii)}\ \frac{d}{dt}\left(v_3\right) = -v_1 Gu$$

$$\text{(iv)}\ \frac{dv_4}{dt} = u$$

Where,

$$\rho = \rho_o - \frac{v_3}{v_1}\frac{\rho_1}{g}$$

$$u = \frac{v_3}{v_1 \rho}$$

$$r = \sqrt[3]{v_1}$$

The system of differential equations (3-2) are now in a suitable form to be solved with a numerical methods solution. The numerical method employed by this thesis is detailed in the next section.

Numerical Methods Solution

The model presented in this thesis uses the mathematical software program, *Mathcad PLUS 6.0 Professional Edition™* to solve the system of differential equations (3-2). The *Mathcad™* template developed to solve for the stabilization height uses a numerical methods solver function included in the *Mathcad™* software. In particular, the *Mathcad™* function "rkfixed" is used to solve for the variables v_1, v_2, v_3, and v_4. The function "rkfixed" is a Runge-Kutta numerical methods solving algorithm. This function requires a vector of initial values for v, endpoints of the interval to be solved, and the step size used to progress through the interval.

Assumptions

To determine a vector of initial values a series of assumptions are required. The first assumption is that although the ground cloud is assumed to be instantaneous an initial radius is required. The assumption of an instantaneous cloud, taken strictly, would result in the cloud having no initial volume and therefore an infinite mass. This is not realistic in real world applications, as a point of infinite mass would never rise off the ground. Therefore the assumption is that the rocket launch creates an initial ground cloud in the first few seconds of the launch, as the rocket leaves the pad and gains sufficient altitude it soon has only a negligible affect on the ground cloud. It is at this point in the launch that the assumption of an instantaneous cloud is made and as such the cloud has an initial volume. It is also assumed that the cloud can be approximated as a sphere near the ground and through its rise, and therefore an initial radius (r_o) is used to characterize this volume of the cloud. The next assumption is that the ground cloud has no initial vertical velocity. This seems appropriate as the cloud is not imparted with any mechanical force thrusting it upward off the ground. Only the buoyant force created by the density difference between the cloud and the ambient air provide any vertical force on the cloud. The next assumption concerns the change of the ambient air density with height and is required because the template can not be iterated from the ground level through a series of intermediate elevations. This results in the need to establish a characteristic ambient air density change ($d\rho/dz$) as a function of height. The final assumption concerns the value of the coefficient of entrainment (α). Original testing of the model uses the same assumption as the REEDM used with an estimate of 0.64. With the above assumptions

the model has all the required inputs and is used to predict the stabilization height of the ground cloud. Further experiments are then performed to better determine approximations for the initial ground cloud radius (r_o), density change rate ($d\rho/dz$), and coefficient of entrainment (α).

<u>Predictive Differential Equation Model</u>

The basic differential equation model discussed previously is based on meteorological and observational data for the five CCAS launches. The purpose of the thesis is to develop a model which can be used to predict stabilization heights for future launches. In order to accomplish this, the model will have to determine appropriate input parameters for the initial radius, density rate change, and coefficient of entrainment based solely on information known prior launch. Therefore, observational data used in the basic model will not be available for use to determine these input parameters in the predictive model. The following section therefore, details the methodology employed to determine appropriate values for the three input parameters based only on meteorological data which is known prior to launch.

Coefficient of Entrainment Determined from Observation Data

A better approximation for the coefficient of entrainment (α) values is desired as it is unlikely that a default value of 0.64 is appropriate for all launch conditions. Therefore, values for the coefficient of entrainment are calculated based on observation data of the CCAS launches. The data of interest is the cloud stabilization height and the radius of the cloud at that height. With this information and assuming an initial radius of 200 meters,

equation (2-50) is used to solve for the coefficient of entrainment (α). Using this methodology a value for the coefficient of entrainment is determined for each of the CCAS launches. These α values, referred to as observed coefficients values, are then used to determine better values for the other two input parameters.

Initial Cloud Radius Determination

One of the three input parameters required to run the model is the initial cloud radius. An experiment with the model is accomplished in order to determine an appropriate value for this parameter. This is accomplished using the observed coefficient of entrainment values and a value for the density rate change ($d\rho/dz$) calculated from meteorological data taken before the launches. The value of ($d\rho/dz$) is calculated using equation (3-3), where the subscript (stab) refers to values at the stabilization height and the subscript (o) refers to ground level values.

$$\frac{d\rho}{dz} = \frac{\rho_{stab} - \rho_o}{z_{stab} - z_o} \qquad (3\text{-}3)$$

With appropriate values for the coefficient of entrainment and the density rate change the model is run a number of times to determine an appropriate value of the initial radius. The appropriate value will be chosen based on the initial radius which provides the most accurate stabilization heights for all the CCAS launches.

Determining a Characteristic Density Rate Change Value

Once an appropriate value for the initial radius has been determined the next step is to determine a method for predicting a reasonable value for the density rate change as a function of altitude. Initial values for the density rate change ($d\rho/dz$) were based on the density change between the density values at the stabilization height and the ground. If the model is to be a predictive tool then this methodology for determining ($d\rho/dz$) is unacceptable as the stabilization height is an unknown before the launch occurs. Towards this end an experiment is conducted to determine if a characteristic density rate change can be determined. The experiment will use a density rate change between a set altitude and the ground for all CCAS launches to determine if the model provides reasonable stabilization height predictions. The results of this experiment will provide a way to approximate a reasonable characteristic density rate change value that may be determined before launch. This will allow the model to be used as a predictive tool.

Determining a Coefficient of Entrainment Value

At this part in the methodology two of the three input parameters are determined. Namely the values for an initial radius and a characteristic density rate change. The final input parameter to be determined is the coefficient of entrainment value (α). It seems reasonable to assume that the coefficient of entrainment is a function of meteorological conditions. Therefore, an investigation of meteorological conditions is conducted to determine a predictive tool for the coefficient of entrainment.

Meteorological conditions have a major impact on the vertical motion experienced by a rocket exhaust ground cloud. Variables such as potential temperature lapse rate, dew point temperature, relative humidity, wind speed, and wind direction all can have an impact on the rise of buoyant clouds. An investigation of these variables can lend insight into the type and magnitude of these effects. The analysis is conducted using the rawinsonde data collected at each of the five launches from CCAS. The analysis includes profiling the variables of interest with height, and determining statistical mean and variance values for regions of interest in altitude. The first region includes the elevations near the ground typically beginning about 75 meters off the ground to approximately 400m. This region is important because it represents the elevations of initial cloud development and movement. Effects of air entrainment and atmospheric stability at these initial elevations seem to have a strong effect on cloud behavior. The second region extends up from a height near the ground to the elevation of cloud stabilization. This second region is of interest to ensure that anomalies of atmospheric conditions at higher elevations are not overlooked. The analysis consists of comparing the profiles, means and variances for the variables of interest to the observed behavior of the ground cloud. This comparison helps to highlight which variables seem to have an effect on entrainment and therefore cloud rise.

The results of the investigation will highlight meteorological conditions which seem to have the greatest affect on the coefficient of entrainment. The conditions highlighted are then used to determine a function based on these conditions to predict a coefficient of entrainment value specific to each launch.

Final Run of the Model for CCAS and VAFB Launches

The final step in the process is to run the model using the determined initial radius value, the characteristic density rate change value, and the predictive coefficient of entrainment function. The stabilization height results of these model runs are then compared to observed stabilization height values and predicted values determined using the REEDM and the buoyancy formula. In addition, to help determine the ability of the model to predict stabilization heights at location other than CCAS the model is run for the two VAFB launches. The results of the VAFB are then compared in a similar manner as those for CCAS.

IV. RESULTS

Introduction

The results from this thesis are organized into sections presenting four categories of data. Each section show results of the predicted ground cloud stabilization height and the observed stabilization heights for the five launches at CCAS. The first three sections represent the results determined using approximations of required input variables for the model. The first section shows results of the model experiments performed to determine a reasonable initial ground cloud radius. The second section show results of approximating the density rate change ($d\rho/dz$). The third section shows the cloud stabilization height results using two possible prediction equations for the coefficient of entrainment. This third section also provides the statistical analysis used in determining the regression equations for the coefficient of entrainment. The fourth section shows the results of employing the model with an appropriate initial radius, density rate change, and coefficient of entrainment along with results determined by the REEDM and buoyant formula.

Observed Coefficient of Entrainment Results

After creating the basic model from the conservation equations introduced by Morton et al. (2: 15-16) the first step in the model's development is the determination of an appropriate initial radius. In order to accomplish this, values for the coefficient of entrainment and density rate change as a function of height need to be assumed. The coefficients of entrainment for each of the five CCAS launches are determined based on

observation data provided by summary reports of the launches(7: 16-18; 8: 17-24; 9: 16-18; 11: 11-13; 12: 15-18). Specifically, this is accomplished using data of the cloud top and center line heights as a function of time and is provided in Appendix B. This data is then used to determine the maximum radius of the ground cloud. This information is used to determine a coefficient of entrainment using equation (2-52). Table 4-1 shows the data used in this process and the resulting value for the coefficient of entrainment.

Mission	K-2	K-16	K-19	K-21	K-23
Plume Top (m)	2635	1877	2482	2120	2211
Plume Center (m)	1871	1023	1774	1375	1640
Assumed Initial Radius (m)	200	200	200	200	200
Observed Entrainment (α)	.30	.64	.29	.40	.23

Table 4-1: Initial Coefficient of Entrainment Values

Initial Density Rate Change Values

In order to run the model a second value must be determined. This input to the model is the density change rate as a function height ($d\rho/dz$). This value is determined using a linear difference between the density at the stabilization height, known from observation, and the density at ground level. The values calculated for the density rate change are shown in Table 4-2.

Mission	K-2	K-16	K-19	K-21	K-23
Density Rate Change (kg/m^4)	-9.5E-5	-1.2E-4	-9.9E-5	-1.0E-4	-1.0E-4

Table 4-2: Initial Density Rate Change ($d\rho/dz$) Values

Prediction Results Using Initial Radius and Observed Entrainment Values

With the above values for the coefficient of entrainment and the density rate change the model is used to perform a series of runs using initial ground cloud radii ranging from 100 meters to 250 meters in order to determine an appropriate initial radius for the model. The Mathcad template summarizing this data is included in Appendix C.

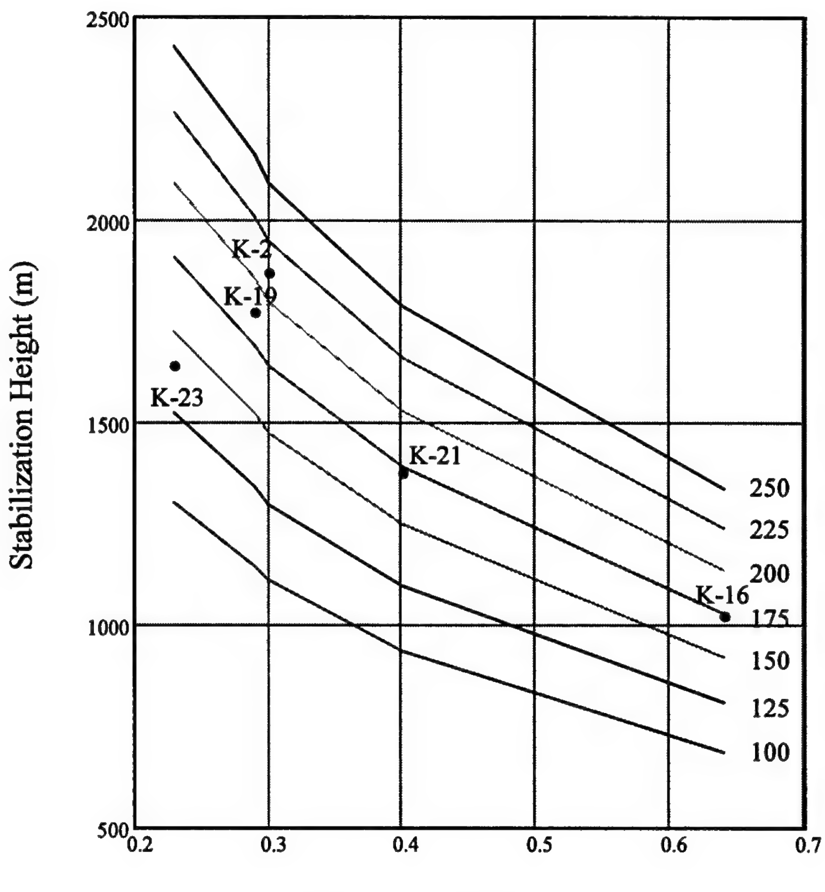


Figure 4-1: Plot of Initial Radius Experiment

The initial radius of 175 meters seems to be an appropriate value for use in the model based on the data represented in Figure 4-1.

Stabilization Height Results Using a Characteristic Density Rate Change

The atmospheric density rate change is the second key input required to use the model. In order to predict this value it is assumed that the change in atmospheric density with height can be approximated using the change in density from an altitude of approximately 100 meters to an altitude of 1000 meters. With this assumption the model is run for each of the five CCAS launches to determine stabilization heights. In addition, the initial radius is again varied in length with a narrower range including radii from of lengths of 150 meters to 230 meters in length. The results of this effort are summarized in Figure 4-2 with the raw data presented in Appendix D.

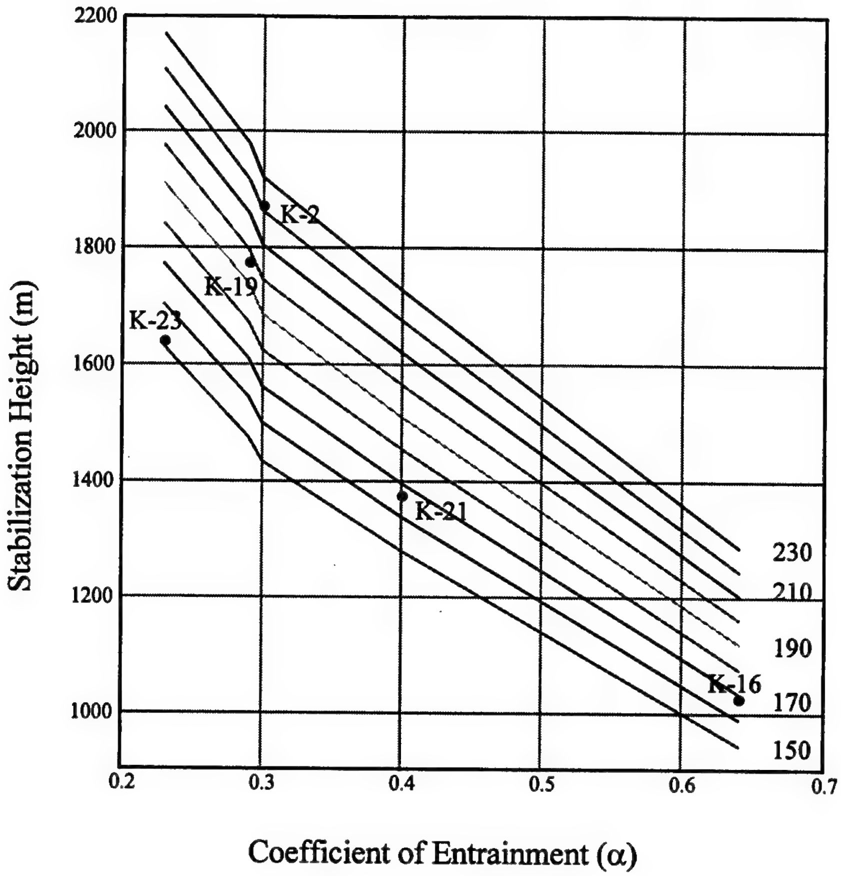


Figure 4-2: Plot of Stabilization Heights with Change of Density Assumption

The above figure shows little change in the prediction of ground cloud stabilization heights due to basing the density rate change on atmospheric conditions from approximately 100 meters to 600 meters. This is evidence that the characteristic density rate change value is a good approximation of the value based on knowing the observed stabilization height. In addition, the previous use of an initial radius of 175 meters seems close, although 170 meters seems more appropriate. Based on the data illustrated in Figure 4-2 the density rate change seen in the 100 to 600 meter range and an initial radius of 170 is used in the following work with the model.

Results of Determining an Appropriate Coefficient of Entrainment

The third and final key input required for use of the model is the coefficient of entrainment. As discussed in Chapter 2 the rate of entrainment is affected by various atmospheric conditions. This sections illustrates the atmospheric variables which seem to have the most significant impact on entrainment. Atmospheric parameters such as potential temperature lapse rates, potential temperature, temperature, dew point temperature, wind speed, wind direction, and relative humidity are profiled with height for the five CCAS launches. These profiles are included in Appendix E. Inspection of these profiles indicate that the atmospheric parameters including potential temperature lapse rate, dew point temperature, and wind speed have an impact on entrainment. Therefore, these parameters are used in a statistical analysis to determine a linear regression fit to predict a coefficient of entrainment. This is accomplished by taking mean values for these parameters and fitting them to the calculated entrainment values shown in Table 4-1. Mean values are calculated for these parameters using different regions of data. The regions are used to depict conditions near the ground and conditions extending higher into the atmosphere. The lower regions are used to depict conditions which are assumed to have greater impact on the cloud rise. This assumption is made as effects early in the cloud rise process affect the cloud for a greater amount of time. For instance, if the cloud experiences an unstable atmosphere close to the ground it may be pulled apart before it has a chance to rise into a more stable atmospheric condition. In addition to the mean values calculated for the potential temperature lapse rates a weighted

average of this parameter is also determined for the low and high regions. A summary of this data and coefficient of entrainment values are presented in Table 4-3.

Mission	Altitude (m)	K-2	K-16	K-19	K-21	K-23
Low Altitude Stability Factor (°C)	75 - 300	0.20	-0.02	1.42	-0.19	1.36
High Altitude Stability Factor (°C)	300 - 2000	0.38	0.37	0.76	0.42	-1.72
Lapse Rate (°C/m 10^{-3})	100 - 600	-2.62	2.41	6.00	-1.11	8.28
Dew Point (°C)	100 - 600	21.5	15.2	20.2	16.3	21.4
Wind Speed (m/s)	100 - 600	6.23	7.12	7.42	3.48	4.08
Observed Entrainment (α)		0.32	0.67	0.30	0.42	0.24

Table 4-3: Mean and Weighted Average Meteorological Data

The data shown in Table 4-3 is then investigated to determine a reasonable linear regression fit to predict coefficient of entrainment values. The regression analysis is performed using a statistics package and is provided in Appendix F. The following two regression equations are determined to best fit the data.

$$\alpha_{pred1} = 1.05 - .039 * sf - .043 * dp + .031 * ws \tag{4-1}$$

$$\alpha_{pred2} = 1.21 - 0.039 * hsf - 0.056 * dp + 0.042 * ws \tag{4-2}$$

Where:

sf = low altitude stability factor

hsf = high altitude stability factor

dp = mean dew point temperature

ws = mean wind speed

The adjusted R-squared values for equation (4-1) and (4-2) are 0.656 and 0.688 respectively.

The regression equations are then used to predict coefficient of entrainment values for the five CCAS launches. A mean square difference is calculated comparing the predicted results to the observed coefficient of entrainment values. These results are summarized in Table 4-4 along with results based on the REEDM default value for the coefficient of entrainment.

Mission	MSD	K-2	K-16	K-19	K-21	K-23
$\alpha_{observed}$	N/A	0.32	0.67	0.30	0.42	0.24
α_{pred1}	0.018	0.31	0.62	0.36	0.46	0.20
α_{pred2}	0.017	0.26	0.65	0.36	0.43	0.25
α_{REEDM}	0.136	0.64	0.64	0.64	0.64	0.64

Table 4-4: Predicted Coefficient of Entrainment Values

Results Using the Predicted Coefficient of Entrainment Values

The predicted coefficient of entrainment values determined are then entered into the model for the five CCAS launches and ground cloud stabilization heights are determined. The results from this exercise are provided in Figures 4-3 and 4-4.

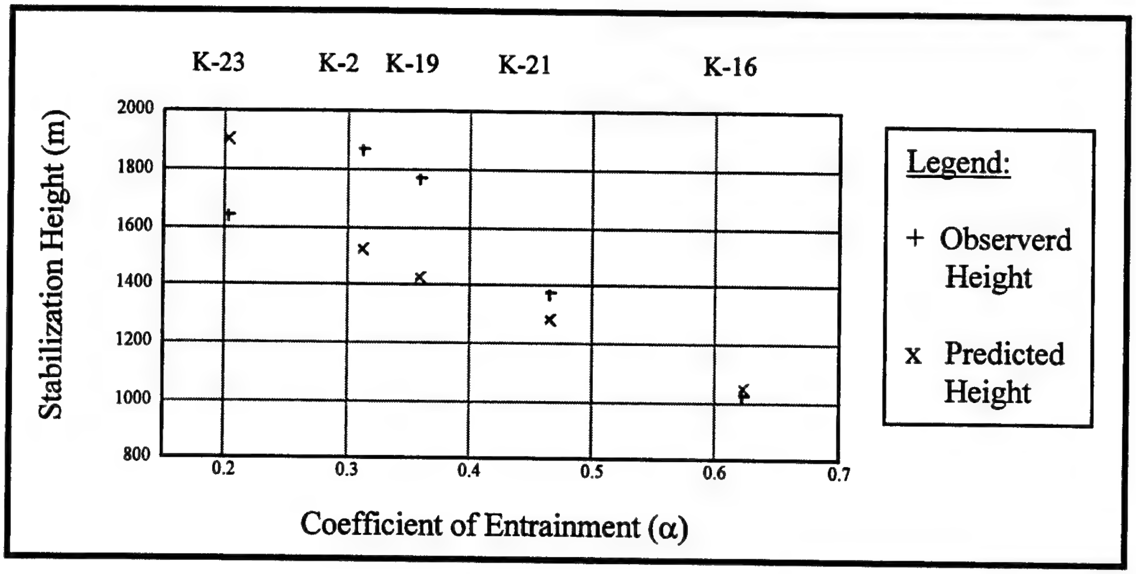


Figure 4-3: Stabilization Height Predictions using α_{pred1}

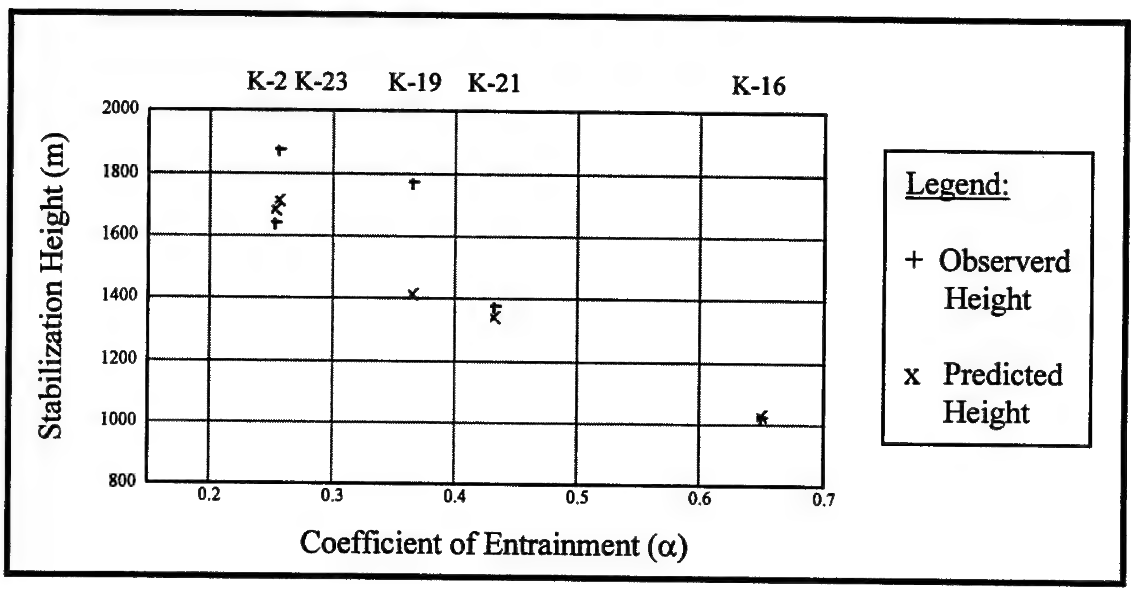


Figure 4-4: Stabilization Height Predictions using α_{pred2}

Comparison of Final Model Results with Previous Work

With appropriate values determined for the initial radius, density rate change of the atmosphere, and coefficient of entrainment the model is complete. This section compares the results of the model with previous stabilization height predicted results. The previously determined results include those determined by the REEDM and the buoyant formula produced by Sand (3: 34). A summary of these results are shown in Figures 4-5 and 4-6 along with the models results using the two coefficient of entrainment predictions and the observed stabilization heights for the five launches at CCAS.

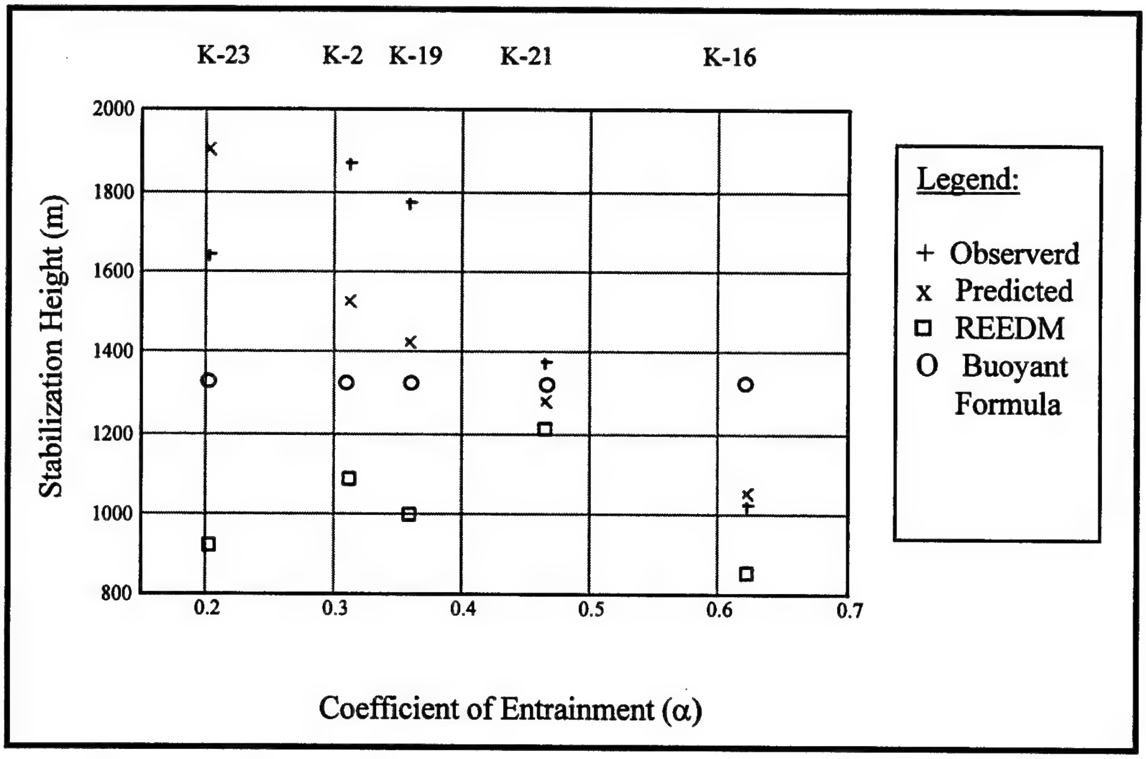
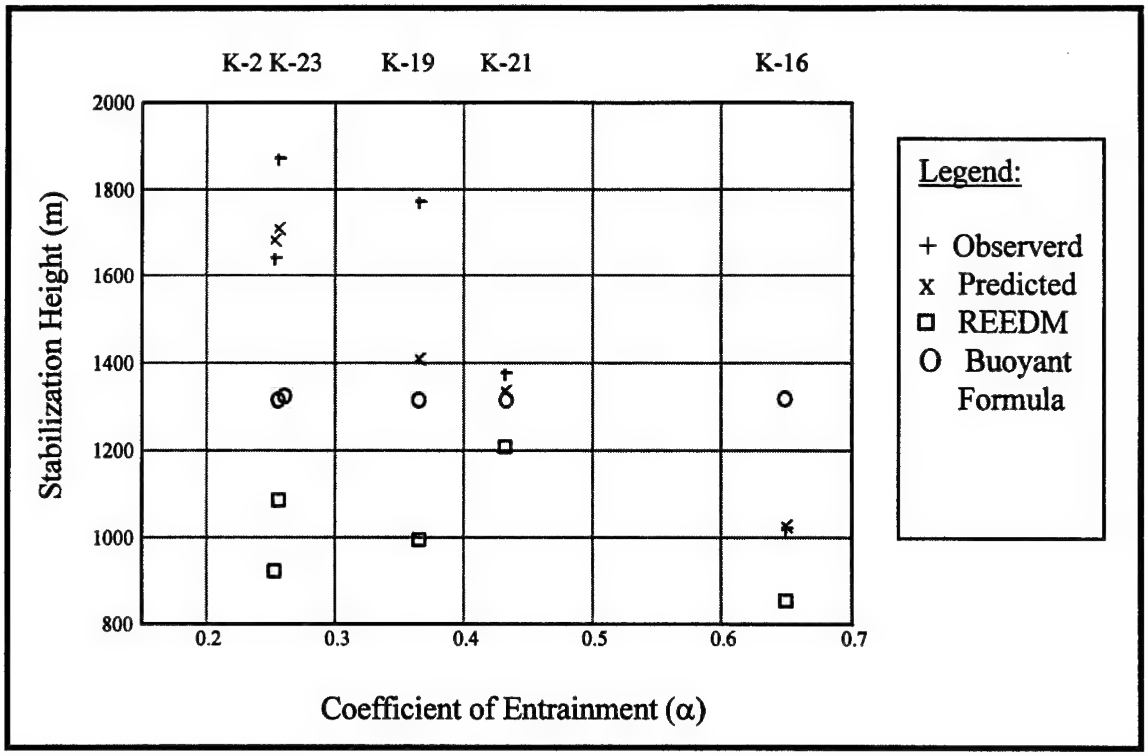


Figure 4-5: Summary of Stabilization Heights Using α_{pred1}



Note: Buoyant Formula results for launch K-2 are assumed from other launches.

Figure 4-6: Summary of Stabilization Heights Using α_{pred2}

Mean square difference calculations are performed comparing stabilization height predictions to the observed stabilization heights and are provided in Appendix F. The buoyant formula results are not used as it does not appear to account for the fluctuation of stabilization heights that are observed. A summary of these findings along with the stabilization heights for the CCAS launches are included in Table 4-5.

Mission	MSD	K-2	K-16	K-19	K-21	K-23
Observed (m)	N/A	1871	1023	1774	1375	1640
Model Pred 1 (m)	110	1526	1051	1424	1280	1904
Model Pred 2 (m)	80	1710	1027	1410	1338	1682
REEDM (m)	270	1087	854	997	1211	923

Table 4-5: Summary of Ground Cloud Stabilization Height Data for CCAS Launches

Results for VAFB Launches

To this point, in the thesis, only CCAS launch data have been used in the analysis. In an attempt to determine the ability of the model to predict stabilization heights occurring at different locations, an analysis using the two VAFB launches is performed. Table 4-5 summarizes the coefficient of entrainment predictions and the observed coefficient of entrainment values.

Mission	K-15	K-22
Observed	0.52	0.56
Model Pred 1	0.84	0.93
Model Pred 2	0.96	1.16
REEDM	0.64	0.64

Table 4-6: Coefficient of Entrainment Values for VAFB Launches

Table 4-6 summarizes the stabilization height predictions determined by the model along with observed and REEDM data.

Mission	K-15	K-22
Observed (m)	658	702
Model Pred 1 (m)	714	704
Model Pred 2 (m)	704	630
REEDM	514	424

Table 4-6: Summary of Ground Cloud Stabilization Height Data for VAFB

V. CONCLUSIONS

Conclusions Introduction

The model presented in this thesis more accurately predicts rocket exhaust cloud stabilization heights than the REEDM or buoyant formula prediction models for the rocket launches investigated at CCAS and VAFB. It is important to note however, that the model was developed only with data from the five CCAS launches. An attempt to measure the models applicability to differing site conditions was accomplished using the two launches at VAFB. The results proved promising, however with only two launches to test the models robustness it is premature to state that the model is appropriate to predict stabilization heights at different launch facilities. The dearth of data was a limiting factor in the models development and its accuracy may suffer if applied to launch conditions significantly different than those found at CCAS or VAFB. Meteorological conditions seem to have a significant effect on cloud rise and this model is only a small step towards understanding these effects. The remainder of this thesis will discuss conclusions concerning the three key input parameter assumptions made in this model as well as some more specific observations concerning the effect of meteorological conditions on rocket exhaust plume rise.

Conclusions Concerning the Initial Radius Assumption

The model presented in this thesis requires three key inputs to predict a cloud stabilization height. The first key input investigated was the initial ground cloud radius. A value of 170 meters was determined to most accurately predict the stabilization heights

for the CCAS launches. This value seems to reasonable to an approximate point based on observations of rocket launches at the rocket launch facilities. However, it is important to note that actual initial cloud radius values may vary from the 170 meters used in this thesis. The assumption of when the ground cloud behaves as an instantaneous cloud is still up for debate. This is critical as the sooner after launch the cloud is assumed instantaneous the smaller the initial radius. Coupled with the sensitivity of the model to the initial radius the predicted stabilization height will vary. A smaller initial radius will lower the stabilization height, while a larger value will increase the stabilization height. For instance as illustrated in Figure 4-1 an increase to the initial radius of 75 meters can raise the stabilization height by approximately 600 meters. It is important to consider this point to understand the model, one of its limitations and the results it produces.

Conclusions Concerning the Change of Atmospheric Density with Height

The model assumes the change in atmospheric density up through the atmosphere may be approximated using a linear rate change from the ground to an elevation of 600 meters. This assumptions appears to be valid based first on the fact that predicted stabilization heights did not vary much from the initial value used. This value being the rate change based on data from the ground to the cloud stabilization height. The second reason for the validity of this assumption is that under most atmospheric conditions the density change with height is reasonably consistent.

Conclusions Concerning the Coefficient of Entrainment Assumption

The model assumes the coefficient of entrainment may be approximated based on a regression analysis. Therefore, the coefficient of entrainment used by the model is based on an empirical analysis encompassing data from only the five CCAS launches. The meteorological parameters the regression analysis employed are the potential temperature lapse rate, dew point temperature, and wind speed. The physical and thermodynamic effects of these parameters in the atmosphere indicate they should have an effect on the rate of entrainment. However, the magnitude of their effects or any synergistic effects that they may cause are still unknown. In addition, there are likely other meteorological parameters which have effects on entrainment and some of them may be significant. Supporting this conclusion are the inflated entrainment values predicted for VAFB launches. However, use of the regression equations does show some improvement over using a single default value. As more launches occur and more data is gathered the ability of this kind of empirical analysis may prove helpful in better predicting coefficient of entrainment values and ground cloud stabilization heights.

Observations Concerning Meteorological Effects on Cloud Rise

Meteorological conditions for the five CCAS launches were investigated in this thesis. Profiles of numerous meteorological parameters were studied to gain understanding in how they may effect rocket exhaust cloud rise. This section will relate some of these observations which may have an impact on cloud growth and rise.

The rocket exhaust ground clouds grow in size principally through the phenomena of entrainment. As discussed in Chapter 2 an unstable atmosphere tends to increase cloud growth. Therefore, atmospheric stability was investigated for different regions of the atmosphere. Low level instability seems to quickly cause the ground cloud to grow rapidly and as such the cloud losses a significant amount of energy in these low levels. This seems to limit the heights these clouds can reach. Low level stability seems to have the opposite effect on the cloud. In essence, the low level stability holds the cloud together keeping the buoyant energy in the cloud and helping it propel itself upward. Upper level stability conditions however seem to have the opposite effect. Upper level stability seems to reduce the clouds ability to rise to great elevations. This follows thermodynamic principles as the stability of the atmosphere serves as a braking force on the cloud. The opposite observation was made concerning upper level instability. Based on this it appears that ground clouds reaching the highest elevations experience low level stability and then quickly enter into upper level instability conditions.

A second meteorological condition which seems to have a strong effect on the rise of exhaust clouds is the amount of moisture in the ambient air. The relative humidity and dew point temperature are the two meteorological parameters capturing this condition. The lower both of these values are the lower the observed stabilization heights. An explanation for this could be that in these conditions evaporation of moisture at the clouds surface diminishes the heat and therefore buoyancy of the cloud. This was illustrated in the investigation of meteorological conditions for the launches at both CCAS and VAFB. First a significant difference in relative humidity and dew point

temperature existed between the launches at CCAS and VAFB. The relative humidity and dew point temperature for the VAFB launches were significantly lower than those at CCAS and the stabilization heights at VAFB were also much lower those observed at CCAS. A second possible effect caused by low dew point temperatures is that it may increase the growth rate of the cloud. The center of a rocket exhaust cloud is the warmest and is protected somewhat from the atmosphere by its cooler outer edges. As a result however, the outer edges are not as buoyant as the center and therefore will not rise as fast. In this way the cooler the dew point the cooler the edges of the cloud and the more the edges stretch down from the sides of the cloud. This whole process serves to entrain a greater volume of cooler air into the cloud and thereby reduces the stabilization height. A final way in which this was illustrated was brought about by the wind direction at CCAS during launches. When prevailing winds were directed over land the cloud stabilization heights were relatively low. A reasonable assumption for this observation is that relative humidity and dew point temperature over land are lower than those experienced over water. The warm water of the Atlantic Ocean serves as a source of humidity for air over the water. Based on this assumption it would seem that ground clouds over land will evaporate more rapidly thereby lowering the stabilization heights reached. A second possible explanation for the low heights achieved over land may be related to wind conditions in concert with ground effects. Ground effects tend to cause greater wind shear over land where the terrain is broken up. Over the ocean there is no ground effects and the wind motion is less turbulent. This factor may explain why the two largest observed coefficients of entrainment were experienced by launches with the wind directed inland. Therefore, wind direction may be an important factor to consider when

predicting a coefficient of entrainment for launches. The above discussion provides some hypotheses for how meteorological effects may impact the stabilization height of rocket exhaust ground clouds.

Conclusions Summary

The model presented in this thesis is founded on basic conservation equations, however empirical assumptions have been made concerning key input variables. The empirical assumptions were developed primarily from data collected during five Titan IV rocket launches at CCAS. This is not enough data to make any definite conclusions concerning prediction of ground cloud stabilization heights. However, this thesis does illustrate some possible trends in the effects of meteorological conditions on cloud rise behavior. Additional rocket launch data is required to validate any conclusions garnered from use of this model.

The model was developed to predict the ground cloud stabilization heights as accurately as possible. No concern was given to make the predictions calculated conservative in nature. A model employed in real world scenarios must be concerned with producing conservative results. With this thought in mind the model presented in this thesis can be altered to produce conservative results through a change in the initial radius assumption. In this manner the model can be altered to produce results as conservative as is required.

Recommended Follow-On Research

1) The effort in this thesis to understand meteorological affects on entrainment can be further studied based on more detailed application of physical and thermodynamic principles.
2) The efforts of this thesis were limited by the amount of available rocket launch data. As more data becomes available from future launches the empirical analysis for the coefficient of entrainment may be a source for further research.
3) The model is currently incapable of predicting cloud stabilization heights in an iterative manner. Future efforts could develop the model allowing for an iterative process to be used and thereby taking advantage of meteorological data to a greater extent.

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APPENDIX A

Rawinsonde meteorological data files for:

K-2
K-15
K-16
K-19
K-21
K-22
K-23

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALT GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	WIND SHEAR (/S)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	16	0.0	0.0	180	2.6	5.0	0.0000	28.40	28.3	25.40	1011.0	84.0
2	300	284.0	86.6	193	3.6	7.0	0.0043	28.49	28.5	24.30	1001.4	78.8
3	600	584.0	178.0	206	4.7	9.2	0.0089	28.58	28.7	23.14	991.2	73.3
4	1000	984.0	299.9	224	6.2	12.0	0.0150	28.70	28.9	21.60	977.7	66.0
5	1300	1284.0	391.4	226	6.2	12.1	0.0114	28.07	28.3	21.00	967.8	65.7
6	1600	1584.0	482.8	229	6.3	12.2	0.0078	27.44	27.8	20.40	958.0	65.4
7	2000	1984.0	604.7	232	6.4	12.4	0.0030	26.60	27.0	19.60	944.8	65.0
8	2300	2284.0	696.2	234	6.3	12.2	0.0027	25.73	26.2	19.21	935.2	67.1
9	2600	2584.0	787.6	236	6.2	12.0	0.0024	24.86	25.4	18.82	925.6	69.2
10	3000	2984.0	909.5	238	6.1	11.8	0.0020	23.70	24.3	18.30	912.8	72.0
11	3300	3284.0	1001.0	241	5.9	11.6	0.0023	22.77	23.4	18.18	903.4	75.6
12	3600	3584.0	1092.4	244	5.8	11.3	0.0026	21.84	22.5	18.06	894.0	79.2
13	4000	3984.0	1214.3	248	5.7	11.0	0.0030	20.60	21.4	17.90	881.5	84.0
14	4300	4284.0	1305.8	252	5.6	10.9	0.0033	19.85	20.6	17.30	872.3	84.9
15	4600	4584.0	1397.2	256	5.5	10.7	0.0036	19.10	19.9	16.70	863.2	85.8
16	5000	4984.0	1519.1	261	5.4	10.5	0.0040	18.10	19.0	15.90	851.0	87.0
17	5300	5284.0	1610.6	265	5.3	10.4	0.0040	17.65	18.5	14.58	842.1	82.5
18	5600	5584.0	1702.0	269	5.3	10.3	0.0040	17.20	18.1	13.26	833.2	78.0
19	6000	5984.0	1823.9	275	5.2	10.1	0.0040	16.60	17.6	11.50	821.3	72.0
20	6300	6284.0	1915.4	278	5.1	9.9	0.0037	16.30	17.3	9.40	812.6	64.8
21	6600	6584.0	2006.8	281	4.9	9.6	0.0034	16.00	17.0	7.30	804.0	57.6
22	7000	6984.0	2128.7	285	4.8	9.3	0.0030	15.60	16.7	4.50	792.5	48.0
23	8000	7984.0	2433.5	289	3.6	7.0	0.0040	14.30	15.4	-0.50	764.5	36.0
24	9000	8984.0	2738.3	302	1.7	3.3	0.0070	13.10	14.3	-4.80	737.4	28.0
25	10000	9984.0	3043.1	1	1.0	2.0	0.0050	11.50	12.7	-3.80	711.1	35.0
26	11000	10984.0	3347.9	20	1.3	2.5	0.0010	9.10	10.1	1.00	685.7	57.0
27	12000	11984.0	3652.7	20	1.3	2.5	0.0100	7.30	8.2	-5.30	660.9	41.0

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALTITUDE GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	368	0.0	0.0	0	4.1	8.0	17.8	19.0	11.2	1002.9	65.0
2	422	54.0	16.5	13	5.1	10.0	16.9	18.3	11.0	1001.0	68.0
3	458	89.5	27.3	10	4.9	9.6	16.4	17.8	10.9	999.7	70.1
4	493	125.0	38.1	7	4.7	9.2	15.8	17.4	10.8	998.5	72.0
5	533	164.5	50.1	3	4.9	9.6	15.7	17.4	11.0	997.0	73.6
6	572	204.0	62.2	358	5.1	10.0	15.6	17.4	11.2	995.6	74.8
7	620	252.0	76.8	350	5.9	11.5	15.5	17.5	11.4	993.9	76.8
8	668	300.0	91.4	341	6.7	13.0	15.3	17.5	11.6	992.2	78.3
9	716	348.0	106.1	346	8.3	16.1	15.2	17.5	11.8	990.5	80.0
10	821	453.0	138.1	346	10.8	20.9	17.9	20.8	13.4	986.8	75.0
11	923	554.5	169.0	350	10.3	20.0	18.0	21.2	13.1	983.2	72.9
12	1024	656.0	199.9	353	9.9	19.2	18.2	21.6	12.8	979.7	71.1
13	1188	820.0	249.9	0	8.4	16.3	18.5	22.4	12.4	974.0	67.9
14	1393	1025.0	312.4	17	8.5	16.6	18.8	23.2	11.8	967.0	63.9
15	1516	1148.0	349.9	25	10.2	19.8	18.7	23.5	11.7	962.8	63.6
16	1680	1312.0	399.9	17	7.3	14.2	18.6	23.9	11.5	957.2	63.1
17	1762	1394.0	424.9	29	7.8	15.2	18.6	24.1	11.4	954.4	63.0
18	1844	1476.0	449.9	41	8.4	16.3	18.6	24.4	11.3	951.7	62.6
19	1926	1558.0	474.9	26	7.0	13.7	18.5	24.6	11.2	948.9	62.4
20	2008	1640.0	499.9	11	5.7	11.1	18.5	24.8	11.1	946.1	62.2
21	2518	2150.0	655.3	26	8.9	17.3	18.2	26.0	10.5	929.2	60.7
22	3074	2705.0	824.6	13	8.4	16.4	17.5	26.8	8.8	911.0	57.1
23	3629	3261.0	994.0	360	8.0	15.5	16.7	27.6	7.1	893.1	53.2
24	4198	3830.0	1167.5	358	7.8	15.2	15.4	27.9	6.3	875.1	55.0
25	4768	4400.0	1341.1	356	7.7	14.9	14.0	28.2	5.4	857.4	55.9
26	5326	4958.0	1511.2	351	7.5	14.6	13.8	29.6	3.6	840.3	50.7
27	5884	5516.0	1681.3	346	7.4	14.3	13.6	31.1	1.8	823.5	44.6
28	6470	6102.0	1859.9	332	7.5	14.6	12.1	31.4	2.6	806.2	52.0
29	6927	6559.0	1999.2	319	7.4	14.4	11.7	32.1	-2.5	793.0	37.0
30	7889	7521.0	2292.4	307	6.4	12.5	9.9	33.2	-4.6	765.6	35.6
31	8405	8037.0	2449.7	298	5.9	11.4	9.3	34.2	-5.1	751.2	36.8
32	8921	8553.0	2607.0	290	5.3	10.3	8.6	35.1	-5.6	737.1	35.9
33	9481	9113.0	2777.6	285	5.1	9.8	7.2	35.4	-5.6	722.0	40.6
34	10041	9673.0	2498.3	280	4.8	9.4	5.8	35.7	-5.7	707.2	43.4

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALT GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	16	0.0	0.0	20	6.2	12.0	20.8	21.3	16.3	1018.7	76.0
2	62	45.8	13.9	9	6.4	12.5	20.6	21.2	16.3	1017.1	76.4
3	108	91.5	27.9	359	6.7	13.0	20.4	21.1	16.3	1015.4	77.3
4	153	137.3	41.8	348	6.9	13.5	20.2	21.1	16.3	1013.8	78.3
5	199	183.0	55.8	337	7.2	14.0	20.0	21.0	16.3	1012.2	79.0
6	285	268.5	81.8	344	7.2	14.0	19.7	21.0	16.3	1009.1	80.9
7	370	354.0	107.9	351	7.2	14.0	19.5	21.0	16.4	1006.1	82.5
8	456	439.5	134.0	358	7.2	14.0	19.2	21.0	16.4	1003.0	84.2
9	541	525.0	160.0	5	7.2	14.0	18.9	21.0	16.5	1000.0	86.0
10	694	678.0	206.7	7	7.3	14.1	18.4	21.0	16.4	994.6	88.0
11	847	831.0	253.3	8	7.3	14.3	18.0	21.0	16.3	989.2	90.0
12	1000	984.0	299.9	10	7.4	14.4	17.5	20.9	16.2	983.9	92.0
13	1114	1098.0	334.7	11	7.2	14.0	17.2	21.0	16.1	979.9	93.0
14	1389	1372.5	418.3	11	7.2	14.0	17.1	21.5	14.3	970.4	83.6
15	1663	1647.0	502.0	10	7.2	14.0	17.0	22.0	12.5	961.0	75.0
16	2000	1984.0	604.7	4	6.5	12.6	16.2	22.4	13.8	949.5	86.0
17	2226	2210.0	673.6	359	6.2	12.0	15.7	22.7	14.8	941.9	94.0
18	2613	2597.0	791.6	346	6.1	11.9	15.8	24.0	14.6	929.0	93.2
19	3000	2984.0	909.5	332	6.1	11.9	15.8	25.2	14.5	916.2	91.0
20	3494	3478.0	1060.1	317	6.2	12.0	15.4	26.3	13.8	900.0	90.0
21	3862	3846.0	1172.3	304	6.2	12.0	15.0	27.0	13.4	888.4	90.0
22	4000	3984.0	1214.3	301	5.9	11.5	14.8	27.2	13.2	884.0	90.0
23	4500	4484.0	1366.7	290	6.2	12.0	13.9	27.7	12.3	868.3	90.2
24	5000	4984.0	1519.1	278	6.5	12.6	13.0	28.2	11.4	852.8	90.0
25	5082	5066.0	1544.1	278	6.7	13.0	12.9	28.4	11.3	850.0	90.0
26	5971	5955.0	1815.1	277	7.7	15.0	11.4	29.4	9.4	823.4	88.0

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALT GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	16	0.0	0.0	170	1.5	3.0	25.1	26.9	22.8	1014.8	87.0
2	57	40.6	12.4	187	2.6	5.0	25.3	27.3	22.9	1013.4	86.6
3	97	81.2	24.7	204	3.6	7.0	25.4	27.5	22.9	1012.0	86.1
4	138	121.8	37.1	221	4.6	9.0	25.6	27.8	23.0	1010.5	85.6
5	178	162.4	49.5	238	5.7	11.0	25.7	28.1	23.0	1009.1	85.1
6	219	203.0	61.9	255	6.7	13.0	25.9	28.5	23.1	1007.7	85.0
7	293	277.3	84.5	255	6.7	13.0	26.0	28.7	22.7	1005.1	81.9
8	368	351.7	107.2	255	6.7	13.0	26.1	28.9	22.2	1002.6	79.3
9	442	426.0	129.8	255	6.7	13.0	26.2	29.2	21.8	1000.0	77.0
10	533	517.0	157.6	255	6.9	13.5	26.4	29.5	21.3	996.9	73.6
11	624	608.0	185.3	255	7.2	14.0	26.5	29.9	20.7	993.8	71.0
12	812	796.0	242.6	255	7.4	14.3	26.0	29.9	20.4	987.4	71.1
13	1000	984.0	299.9	254	7.5	14.6	25.6	30.0	20.1	981.0	72.0
14	1309	1292.7	394.0	251	7.8	15.1	24.9	30.2	19.3	970.6	71.2
15	1617	1601.3	488.1	249	8.0	15.5	24.3	30.3	18.6	960.2	70.6
16	1926	1910.0	582.2	246	8.2	16.0	23.6	30.5	17.8	950.0	70.0
17	2000	1984.0	604.7	245	8.2	16.0	23.5	30.6	17.5	947.7	69.0
18	2500	2484.0	757.1	239	8.7	17.0	22.5	31.0	16.3	931.3	68.0
19	3000	2984.0	909.5	232	9.3	18.0	21.6	31.4	15.1	915.2	66.0
20	3472	3456.0	1053.4	228	9.8	19.0	20.8	32.0	14.8	900.0	68.0
21	4000	3984.0	1214.3	224	10.0	19.5	19.8	32.5	13.9	883.6	69.0
22	5000	4984.0	1519.1	218	10.3	20.0	17.7	33.4	13.3	852.9	76.0
23	5088	5072.0	1545.9	218	10.3	20.0	17.4	33.4	13.1	850.0	76.0
24	5602	5586.0	1702.6	216	10.3	20.0	16.4	33.8	11.9	834.9	75.0
25	6000	5984.0	1823.9	215	10.4	20.2	15.8	34.3	10.8	823.1	73.0
26	6153	6137.0	1870.6	214	10.3	20.0	15.5	34.4	10.4	818.6	72.0
27	6782	6766.0	2062	214	10.3	20.0	14.1	34.7	7.4	800.0	64.0
28	7000	6984.0	2129	214	10.1	19.7	13.7	34.8	6.4	794.1	61.0
29	7500	7484.0	2281	215	9.7	18.9	13.0	35.4	2.6	779.9	49.7
30	8000	7984.0	2434	216	9.3	18.1	12.4	36.1	-1.3	765.9	40.0
31	8400	8384.0	2555	217	8.7	17.0	12.4	37.0	-6.9	754.9	25.0
32	8561	8545.0	2605	217	8.7	17.0	12.1	37.3	-8.0	750.0	24.0
33	9000	8984.0	2738	218	8.2	15.9	11.3	37.7	-10.5	738.6	21.0
34	9500	9484.0	2891	220	7.9	15.4	10.4	38.3	-11.3	725.2	21.5

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALT GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	16	0.0	0.0	20	1.5	3.0	17.3	17.3	15.0	1021.8	86.0
2	65	49.0	14.9	32	2.1	4.0	18.7	19.0	15.9	1020.0	84.0
3	114	98.0	29.9	45	2.6	5.0	20.0	20.6	16.9	1018.3	81.8
4	163	147.0	44.8	57	3.1	6.0	21.4	22.3	17.8	1016.5	79.7
5	212	196.0	59.7	69	3.6	7.0	22.8	24.0	18.7	1014.8	78.0
6	317	300.7	91.7	70	3.6	7.0	22.5	23.9	18.1	1011.1	76.2
7	421	405.5	123.6	71	3.6	7.0	22.2	23.8	17.5	1007.4	74.7
8	526	510.2	155.5	72	3.6	7.0	21.9	23.7	16.9	1003.7	73.3
9	631	615.0	187.5	73	3.6	7.0	21.6	23.7	16.3	1000.0	72.0
10	816	799.5	243.7	74	3.7	7.1	21.1	23.7	15.9	993.6	72.5
11	1000	984.0	299.9	74	3.7	7.2	20.6	23.7	15.6	987.2	73.0
12	1333	1317.3	401.5	77	3.5	6.7	19.5	23.6	15.2	975.6	76.3
13	1667	1650.7	503.1	79	3.2	6.3	18.5	23.5	14.9	964.2	79.6
14	2000	1984.0	604.7	82	3.0	5.8	17.4	23.4	14.5	652.9	83.0
15	2083	2067.0	630.0	84	3.1	6.0	17.1	23.4	14.5	950.0	85.0
16	2261	2245.0	684.3	87	2.6	5.0	16.5	23.3	14.5	944.1	88.0
17	2832	2816.0	858.3	106	2.1	4.0	15.0	23.5	14.3	925.0	96.0
18	3000	2984.0	909.5	115	1.9	3.6	14.4	23.3	13.7	919.5	95.0
19	3219	3203.0	976.3	131	1.5	3.0	13.7	23.2	12.9	912.3	95.0
20	3589	3573.0	1089.1	149	1.0	2.0	12.9	23.5	12.0	900.0	95.0
21	3786	3770.0	1149.1	166	1.0	2.0	12.5	23.6	11.6	893.8	94.0
22	3857	3841.3	1170.8	186	1.0	2.0	12.3	23.6	11.5	891.5	94.5
23	3929	3912.7	1192.6	205	1.0	2.0	12.2	23.7	11.3	889.2	94.7
24	4000	3984.0	1214.3	225	1.0	1.9	12.0	23.7	11.2	886.9	95.0
25	4091	4074.5	1241.9	241	1.0	2.0	11.8	23.8	11.0	884.0	95.1
26	4271	4255.5	1297.1	274	1.0	2.0	11.4	23.9	10.7	878.2	95.4
27	4907	4891.0	1490.8	304	3.6	7.0	11.4	25.2	2.1	858.2	53.0
28	5161	5145.0	1568.2	303	5.1	10.0	11.7	26.1	-2.4	850.0	39.0
29	6000	5984.0	1823.9	298	8.4	16.3	11.0	27.7	-7.1	824.7	27.0
30	6819	6803.0	2073.6	290	10.3	20.0	9.7	28.9	-9.2	800.0	26.0
31	7195	7179.0	2188.2	287	10.8	21.0	9.4	29.7	-11.8	789.4	21.0
32	8000	7984.0	2433.5	284	11.9	23.2	7.5	30.9	1.7	766.3	67.0
33	8569	8553.0	2607	283	12.3	24.0	7.0	32.4	3.5	750.0	78.0
34	9500	9484.0	2890.7	285	12.9	25.0	5.9	34.1	1.8	725.0	75.3

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALT GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	329.0	-39.0	-11.9	355.0	3.1	6.0	19.4	19.4	10.7	1001.8	57.0
2	383.0	15.0	4.6	0.0	3.1	6.0	18.7	18.7	9.9	999.9	56.0
3	431.0	63.0	19.2	355.0	3.1	6.0	18.1	18.1	9.2	998.2	56.4
4	513.0	145.0	44.2	354.3	3.1	6.0	17.0	17.0	8.0	995.3	56.0
5	620.0	252.0	76.8	359.0	2.6	5.0	16.7	16.7	10.3	991.5	66.0
6	829.0	461.0	140.5	345.0	3.4	6.6	15.0	15.1	9.4	984.1	69.0
7	984.0	616.0	187.8	343.0	4.8	9.3	15.2	15.3	9.8	978.6	70.0
8	1149.0	781.0	238.0	320.0	6.0	11.7	17.9	18.0	9.4	972.9	58.1
9	1262.0	894.0	272.5	295.9	4.9	9.6	19.8	20.0	9.1	969.0	50.0
10	1313.0	945.0	288.0	285.0	4.5	8.7	21.3	21.5	9.9	967.3	48.3
11	1384.0	1016.0	309.7	289.3	5.0	9.7	23.4	23.6	11.1	964.8	46.0
12	1477.0	1109.0	338.0	295.0	5.6	10.9	23.9	24.2	10.3	961.7	42.5
13	1553.0	1185.0	361.2	323.4	5.9	11.4	24.3	24.6	9.7	959.2	39.6
14	2059.0	1691.0	515.4	328.7	6.7	13.1	26.5	27.0	6.5	942.5	28.0
15	2700.0	2332.0	710.8	333.4	6.9	13.4	25.8	26.4	4.5	921.8	25.3
16	3884.0	3516.0	1071.7	340.1	5.5	10.7	23.9	24.8	0.7	884.6	21.6
17	5072.0	4704.0	1433.8	1.9	3.1	6.1	20.9	21.9	-1.2	848.5	22.7
18	6290.0	5922.0	1805.0	34.3	1.7	3.3	18.1	19.2	-1.9	812.7	25.5
19	7538.0	7170.0	2185.4	287.0	0.4	0.8	17.1	18.4	-8.3	777.3	16.8
20	8754.0	8386.0	2556.1	251.2	2.2	4.3	15.2	16.5	-10.0	744.2	16.6
21	10063.0	9695.0	2955.0	245.4	2.8	5.5	13.7	15.1	-11.0	709.9	16.9

MET. LEV. NO.	ALT MSL (FT)	ALT GND (FT)	ALT GND (M)	WIND DIR (DEG)	WIND SPEED (M/S)	WIND SPEED (KTS)	AIR TEMP (DEG C)	AIR PTEMP (DEG C)	AIR DPTEMP (DEG C)	AIR PRES. (MB)	AIR RH (%)
1	16	0.0	0.0	250	2.6	5.0	27.1	29.0	23.6	1016.4	81.0
2	63	46.8	14.3	251	2.7	5.2	26.8	58.8	23.4	1014.8	81.9
3	110	93.6	28.5	253	2.8	5.4	26.5	28.6	23.2	1013.2	82.5
4	156	140.4	42.8	254	2.9	5.6	26.1	28.3	23.1	1011.5	83.2
5	203	187.2	57.1	256	3.0	5.8	25.8	28.1	22.9	1009.9	83.8
6	250	234.0	71.3	257	3.1	6.0	25.5	27.9	22.7	1008.3	84.0
7	329	313.3	95.5	253	3.3	6.3	25.4	28.1	22.6	1005.5	84.5
8	409	392.7	119.7	250	3.4	6.7	25.4	28.2	22.6	1002.8	84.5
9	488	472.0	143.9	246	3.6	7.0	25.3	28.4	22.5	1000.0	85.0
10	659	642.7	195.9	242	3.8	7.3	25.4	29.0	22.3	994.1	83.2
11	829	813.3	247.9	237	3.9	7.7	25.4	29.5	22.1	988.3	81.9
12	1000	984.0	299.9	233	4.1	8.0	25.5	30.1	21.9	982.5	81.0
13	1142	1126.0	343.2	231	4.1	8.0	25.7	30.7	21.6	977.8	78.0
14	1414	1398.0	426.1	229	4.4	8.5	25.5	31.2	20.9	968.7	75.4
15	1686	1670.0	509.0	227	4.6	9.0	25.4	31.8	20.1	959.6	72.0
16	2000	1684.0	604.7	225	5.0	9.7	24.8	32.1	19.8	949.3	74.0
17	2606	2590.0	789.4	225	5.1	10.0	23.7	32.7	18.9	929.6	75.0
18	3000	2984.0	909.5	225	5.0	9.7	22.6	32.8	18.4	916.9	77.0
19	3528	3512.0	1070.5	223	4.6	9.0	21.4	33.1	17.9	900.0	81.0
20	4000	3984.0	1214.3	223	4.0	7.8	20.1	33.2	17.5	885.4	85.0
21	4717	4701.0	1432.9	226	3.1	6.0	18.2	33.3	16.5	863.3	90.0
22	5000	4984.0	1519.1	230	2.5	4.8	17.9	33.7	15.3	854.7	85.0
23	5147	5131.0	1563.9	233	2.1	4.0	17.7	33.9	14.7	850.0	83.0
24	5574	5557.5	1693.9	245	1.6	3.2	17.0	34.3	13.2	837.3	78.6
25	6000	5984.0	1823.9	256	1.2	2.4	16.3	34.7	11.7	824.9	74.0
26	6323	6307.0	1922.4	269	1.0	2.0	15.5	34.8	10.8	815.4	74.0
27	6497	6480.7	1975	289	1.0	2.0	15.3	35.1	9.6	810.2	69.1
28	6670	6654.3	2028	308	1.0	2.0	15.1	35.3	8.3	805.1	64.5
29	6844	6828.0	2081	328	1.0	1.9	14.9	35.5	7.1	800.0	59.0
30	6922	6909.0	2105	342	1.0	1.9	14.8	35.5	6.2	797.9	57.0
31	7000	6984.0	2129	357	1.0	1.9	14.7	35.6	5.3	795.9	54.0
32	7427	7411.0	2259	11	1.0	1.9	14.2	36.0	-0.3	783.8	37.0
33	8628	8612.0	2625	22	1.0	1.9	11.9	37.6	1.6	750.0	49.0
34	9500	9484.0	2891	29	1.0	1.9	10.6	39.0	0.8	727.0	51.5

APPENDIX B

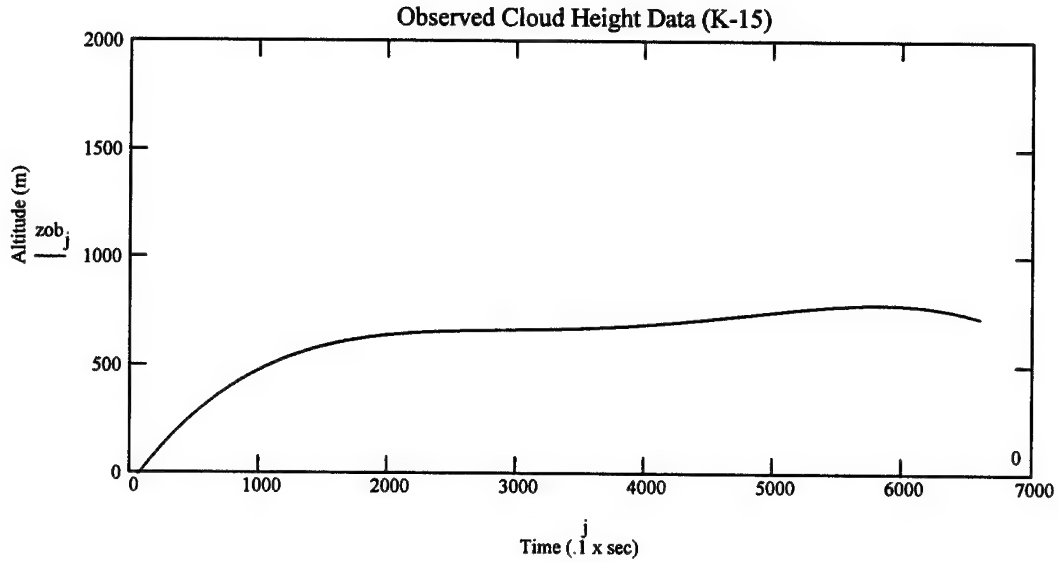
Observed Cloud Height Data:

Observed Cloud Rise Data:

Observed Cloud Rise Data (K-15):

$n := 10 \qquad t_{ob} := 11 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

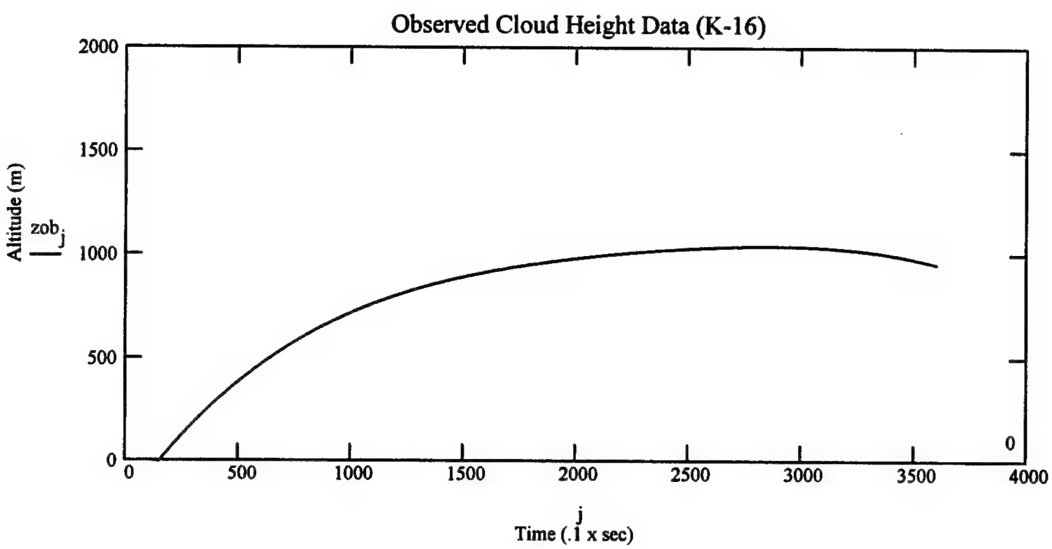
$$zob_j := -0.5498 \cdot (x_j)^4 + 14.152 \cdot (x_j)^3 - 128.34 \cdot (x_j)^2 + 502.52 \cdot x_j - 66.388$$



Observed Cloud Rise Data (K-16):

$t_{ob} := 6 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

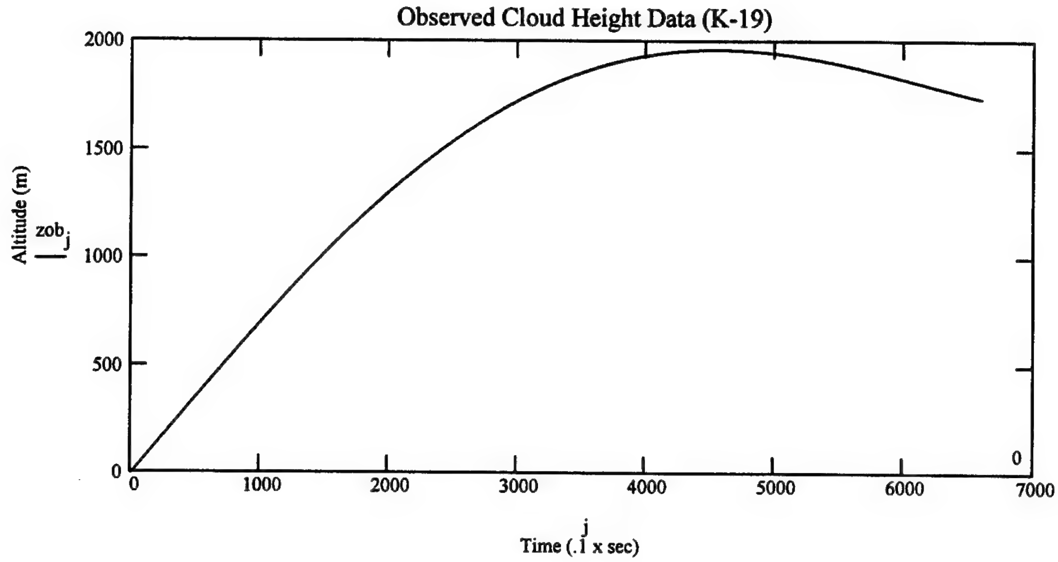
$$zob_j := -3.1435 \cdot (x_j)^4 + 48.496 \cdot (x_j)^3 - 303.52 \cdot (x_j)^2 + 950.75 \cdot x_j - 225.75$$



Observed Cloud Rise Data (K-19):

$n := 10 \qquad t_{ob} := 11 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

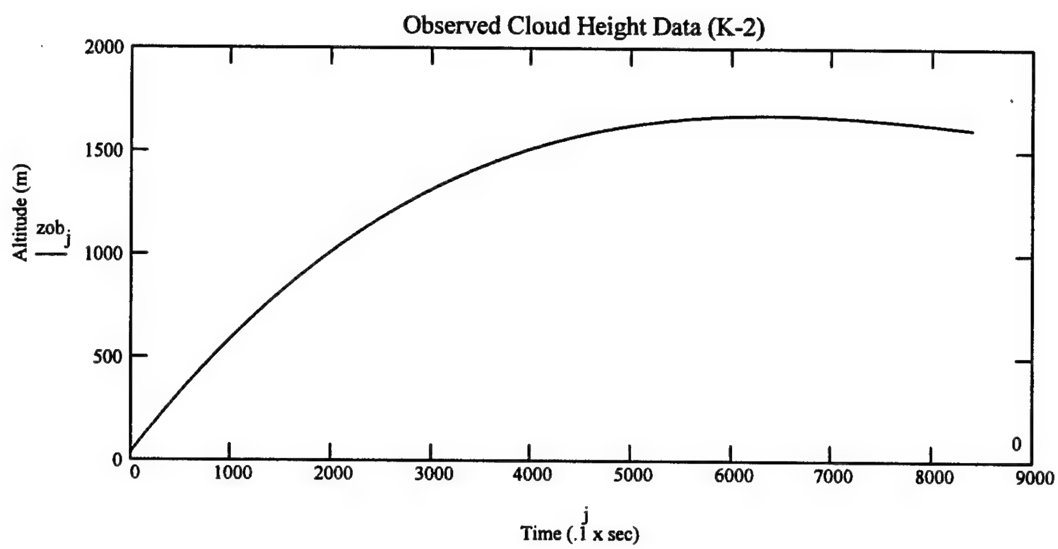
$zob_j := .2675 \cdot (x_j)^4 - 5.643 \cdot (x_j)^3 + 5.1671 \cdot (x_j)^2 + 428.75 \cdot x_j - 12.172$



Observed Cloud Rise Data (K-2):

$t_{ob} := 14 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

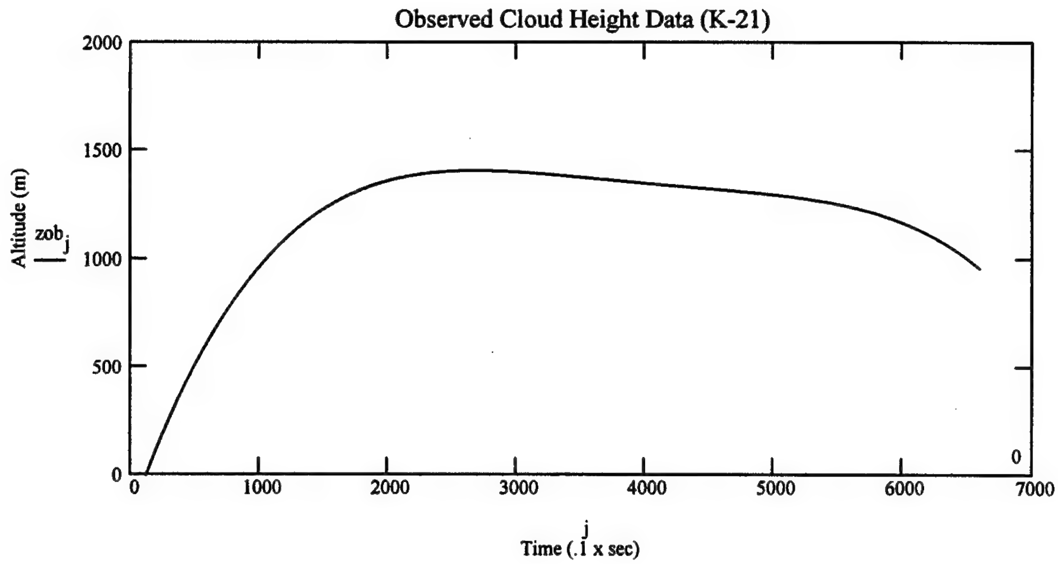
$zob_j := 0.6149 \cdot (x_j)^3 - 27.571 \cdot (x_j)^2 + 377.6 \cdot x_j + 38.08$



Observed Cloud Rise Data (K-21):

$n := 10 \qquad t_{ob} := 11 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

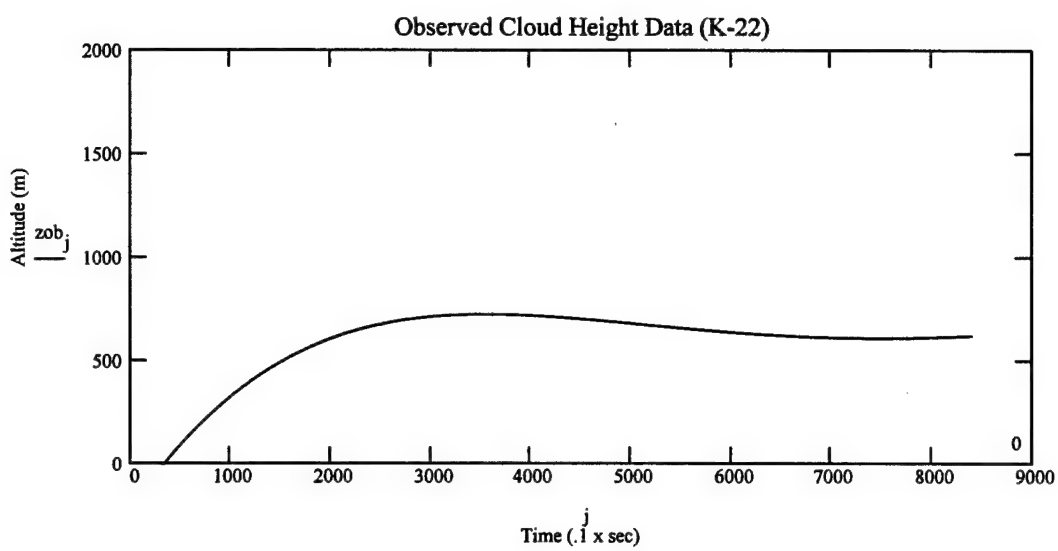
$zob_j := -0.9109 \cdot (x_j)^4 + 24.561 \cdot (x_j)^3 - 245.91 \cdot (x_j)^2 + 1051.9 \cdot x_j - 214.94$



Observed Cloud Rise Data (K-22):

$t_{ob} := 14 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

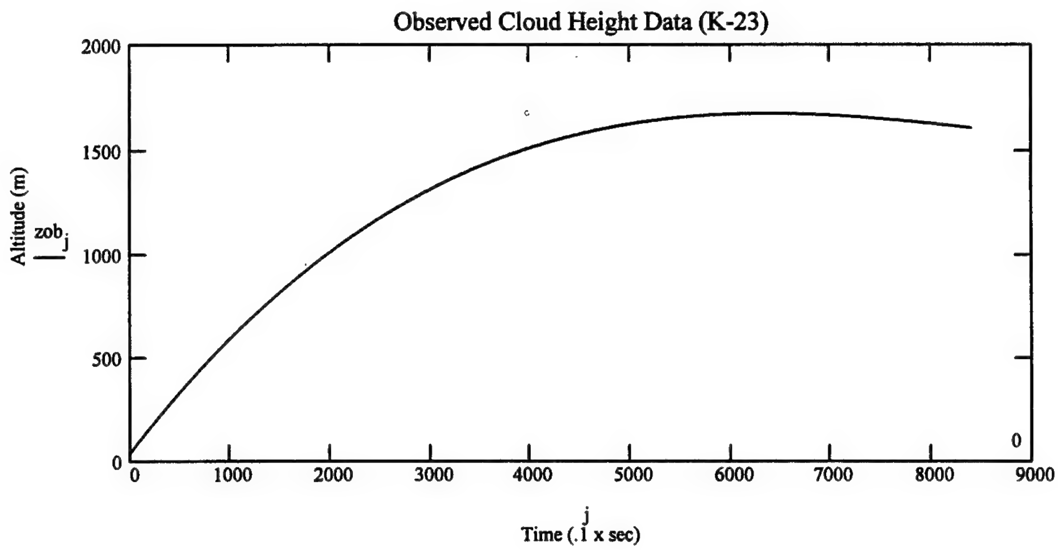
$zob_j := .0016 \cdot (x_j)^5 - .1495 \cdot (x_j)^4 + 4.9361 \cdot (x_j)^3 - 71.141 \cdot (x_j)^2 + 437.92 \cdot x_j - 229.17$



Observed Cloud Rise Data (K-23):

$t_{ob} := 14 \cdot 60 \qquad j := 1 .. t_{ob} \cdot n + 1 \qquad x_j := \frac{j}{60 \cdot n}$

$zob_j := 0.6149 \cdot (x_j)^3 - 27.571 \cdot (x_j)^2 + 377.6 \cdot x_j + 38.08$



APPENDIX C

Mathcad template of initial radius experiment results

Stabilization Height as Function of Initial Cloud Radius:

$\mathrm{ORIGIN} \equiv 1$

$$Kt2 := \begin{bmatrix} 1111 \\ 1300 \\ 1475 \\ 1641 \\ 1798 \\ 1949 \\ 2095 \end{bmatrix} \quad Kt16 := \begin{bmatrix} 685 \\ 806 \\ 921 \\ 1031 \\ 1136 \\ 1238 \\ 1337 \end{bmatrix} \quad Kt19 := \begin{bmatrix} 1147 \\ 1341 \\ 1522 \\ 1693 \\ 1856 \\ 2012 \\ 2162 \end{bmatrix} \quad rads := \begin{bmatrix} 100 & 1111 \\ 125 & 1300 \\ 150 & 1475 \\ 175 & 1641 \\ 200 & 1798 \\ 225 & 1949 \\ 250 & 2095 \end{bmatrix}$$

$$Kt21 := \begin{bmatrix} 936 \\ 1097 \\ 1248 \\ 1392 \\ 1529 \\ 1661 \\ 1788 \end{bmatrix} \quad Kt23 := \begin{bmatrix} 1305 \\ 1522 \\ 1723 \\ 1912 \\ 2092 \\ 2263 \\ 2428 \end{bmatrix}$$

$K2 := Kt2^T \quad K16 := Kt16^T \quad K19 := Kt19^T$

$K21 := Kt21^T \quad K23 := Kt23^T$

$i := 1 .. \mathrm{rows}(K2)$

$A1 := \mathrm{augment}(Kt23, Kt19) \quad A2 := \mathrm{augment}(A1, Kt2) \quad A3 := \mathrm{augment}(A2, Kt21)$

$A4 := \mathrm{augment}(A3, Kt16) \quad alla := A4^T$

$\gamma t := (.23\ \ .29\ \ .3\ \ .4\ \ .64) \quad \gamma := \gamma t^T$

$all := \mathrm{stack}(\gamma t, A4)$

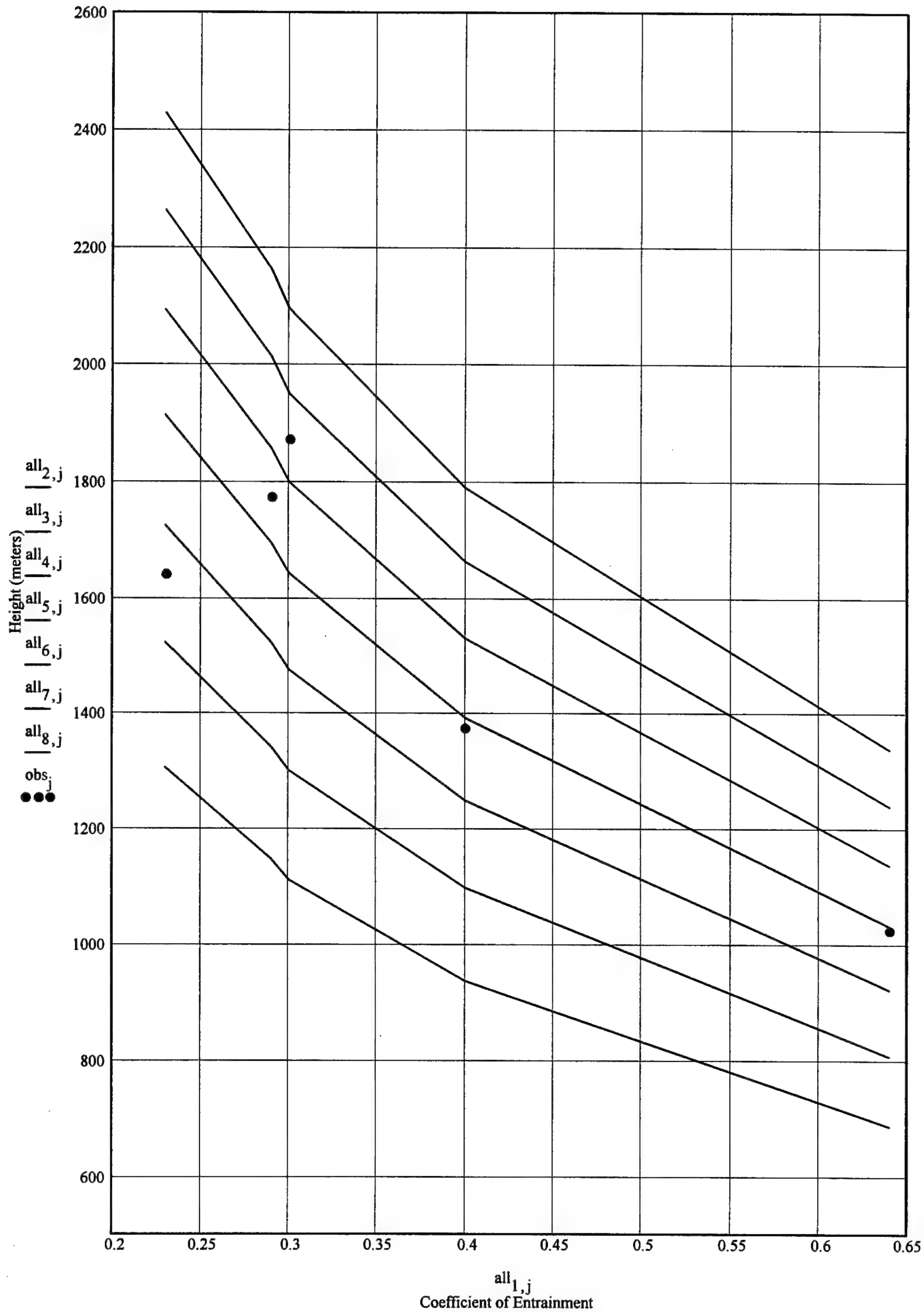
$$all = \begin{bmatrix} 0.23 & 0.29 & 0.3 & 0.4 & 0.64 \\ 1.305 \cdot 10^3 & 1.147 \cdot 10^3 & 1.111 \cdot 10^3 & 936 & 685 \\ 1.522 \cdot 10^3 & 1.341 \cdot 10^3 & 1.3 \cdot 10^3 & 1.097 \cdot 10^3 & 806 \\ 1.723 \cdot 10^3 & 1.522 \cdot 10^3 & 1.475 \cdot 10^3 & 1.248 \cdot 10^3 & 921 \\ 1.912 \cdot 10^3 & 1.693 \cdot 10^3 & 1.641 \cdot 10^3 & 1.392 \cdot 10^3 & 1.031 \cdot 10^3 \\ 2.092 \cdot 10^3 & 1.856 \cdot 10^3 & 1.798 \cdot 10^3 & 1.529 \cdot 10^3 & 1.136 \cdot 10^3 \\ 2.263 \cdot 10^3 & 2.012 \cdot 10^3 & 1.949 \cdot 10^3 & 1.661 \cdot 10^3 & 1.238 \cdot 10^3 \\ 2.428 \cdot 10^3 & 2.162 \cdot 10^3 & 2.095 \cdot 10^3 & 1.788 \cdot 10^3 & 1.337 \cdot 10^3 \end{bmatrix}$$

$ib := 1 .. \mathrm{rows}(all)$

$j := 1 .. \mathrm{cols}(all)$

$i := 2 .. \mathrm{rows}(all)$

$obst := (1640\ \ 1774\ \ 1871\ \ 1375\ \ 1023) \quad obs := obst^T$


2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
Height (meters)
all2,j
all3,j
all4,j
all5,j
all6,j
all7,j
all8,j
obsj
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
all1,j
Coefficient of Entrainment

APPENDIX D

Mathcad template of density rate change experiment

Stabilization Height as Function of Initial Cloud Radius:

ORIGIN ≡ 1

$$Kt2 := \begin{bmatrix} 1387 \\ 1450 \\ 1512 \\ 1573 \\ 1632 \\ 1691 \\ 1749 \\ 1805 \\ 1861 \end{bmatrix} \quad Kt16 := \begin{bmatrix} 920 \\ 965 \\ 1009 \\ 1052 \\ 1094 \\ 1136 \\ 1177 \\ 1218 \\ 1259 \end{bmatrix} \quad Kt19 := \begin{bmatrix} 1438 \\ 1504 \\ 1568 \\ 1631 \\ 1692 \\ 1753 \\ 1812 \\ 1871 \\ 1929 \end{bmatrix} \quad rads := \begin{bmatrix} 150 \\ 160 \\ 170 \\ 180 \\ 190 \\ 200 \\ 210 \\ 220 \\ 230 \end{bmatrix}$$

$$Kt21 := \begin{bmatrix} 1246 \\ 1305 \\ 1362 \\ 1418 \\ 1474 \\ 1528 \\ 1582 \\ 1635 \\ 1688 \end{bmatrix} \quad Kt23 := \begin{bmatrix} 1571 \\ 1641 \\ 1710 \\ 1777 \\ 1842 \\ 1907 \\ 1970 \\ 2033 \\ 2094 \end{bmatrix}$$

$K2 := Kt2^T \quad K16 := Kt16^T \quad K19 := Kt19^T$

$K21 := Kt21^T \quad K23 := Kt23^T$

$i := 1 .. rows(K2)$

$A1 := augment(Kt23, Kt19) \quad A2 := augment(A1, Kt2) \quad A3 := augment(A2, Kt21)$

$A4 := augment(A3, Kt16) \quad alla := A4^T$

$\gamma t := (.23 \;\; .29 \;\; .3 \;\; .4 \;\; .64) \quad \gamma := \gamma t^T$

$all := stack(\gamma t, A4)$

$ib := 1 .. rows(all)$

$j := 1 .. cols(all)$

$i := 2 .. rows(all)$

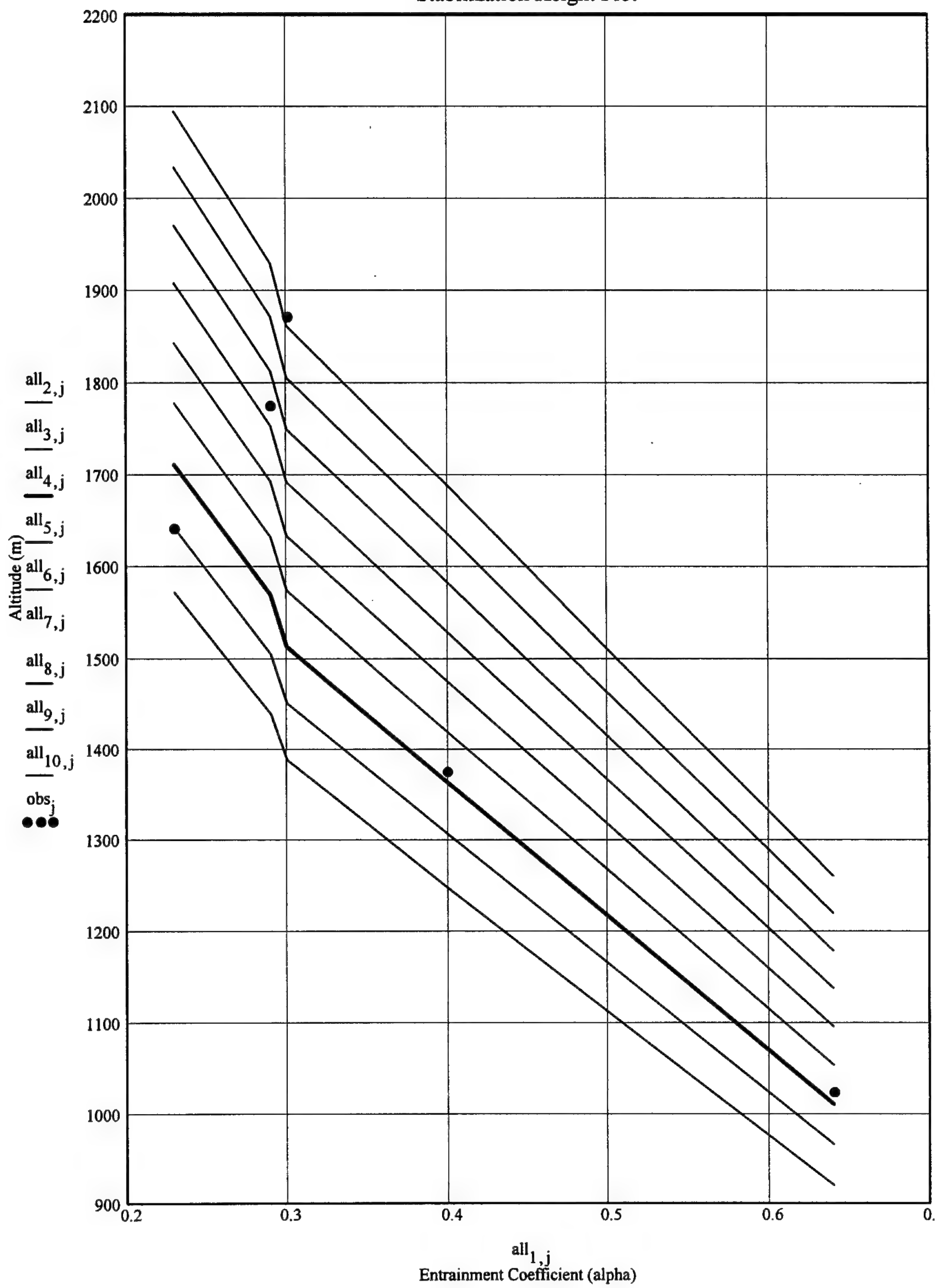
Observed Heights:

$obst := (1640 \;\; 1774 \;\; 1871 \;\; 1375 \;\; 1023) \quad obs := obst^T$

REEDM Predicted Heights:

$RDMt := (923 \;\; 997 \;\; 1087 \;\; 1211 \;\; 854) \quad RDM := RDMt^T$

Stabilization Height Plot



APPENDIX E

Mathcad templates of meteorological profiles

Data:

ORIGIN ≡ 1

$$
d16 := \begin{bmatrix}
5 & 199 & 183.0 & 55.8 & 337 & 7.2 & 14.0 & 20.0 & 21.0 & 16.3 & 1012.2 & 79.0 \\
9 & 541 & 525.0 & 160.0 & 5+360 & 7.2 & 14.0 & 18.9 & 21.0 & 16.5 & 1000.0 & 86.0 \\
12 & 1000 & 984.0 & 299.9 & 10+360 & 7.4 & 14.4 & 17.5 & 20.9 & 16.2 & 983.9 & 92.0 \\
13 & 1114 & 1098.0 & 334.7 & 11+360 & 7.2 & 14.0 & 17.2 & 21.0 & 16.1 & 979.9 & 93.0 \\
15 & 1663 & 1647.0 & 502.0 & 10+360 & 7.2 & 14.0 & 17.0 & 22.0 & 12.5 & 961.0 & 75.0 \\
16 & 2000 & 1984.0 & 604.7 & 4+360 & 6.5 & 12.6 & 16.2 & 22.4 & 13.8 & 949.5 & 86.0 \\
17 & 2226 & 2210.0 & 673.6 & 359 & 6.2 & 12.0 & 15.7 & 22.7 & 14.8 & 941.9 & 94.0 \\
19 & 3000 & 2984.0 & 909.5 & 332 & 6.1 & 11.9 & 15.8 & 25.2 & 14.5 & 916.2 & 91.0 \\
20 & 3494 & 3478.0 & 1060.1 & 317 & 6.2 & 12.0 & 15.4 & 26.3 & 13.8 & 900.0 & 90.0 \\
21 & 3862 & 3846.0 & 1172.3 & 304 & 6.2 & 12.0 & 15.0 & 27.0 & 13.4 & 888.4 & 90.0 \\
22 & 4000 & 3984.0 & 1214.3 & 301 & 5.9 & 11.5 & 14.8 & 27.2 & 13.2 & 884.0 & 90.0 \\
24 & 5000 & 4984.0 & 1519.1 & 278 & 6.5 & 12.6 & 13.0 & 28.2 & 11.4 & 852.8 & 90.0 \\
25 & 5082 & 5066.0 & 1544.1 & 278 & 6.7 & 13.0 & 12.9 & 28.4 & 11.3 & 850.0 & 90.0 \\
26 & 5971 & 5955.0 & 1815.1 & 277 & 7.7 & 15.0 & 11.4 & 29.4 & 9.4 & 823.4 & 88.0
\end{bmatrix}
$$

$$
d19 := \begin{bmatrix}
6 & 219 & 203.0 & 61.9 & 255 & 6.7 & 13.0 & 25.9 & 28.5 & 23.1 & 1007.7 & 85.0 \\
9 & 442 & 426.0 & 129.8 & 255 & 6.7 & 13.0 & 26.2 & 29.2 & 21.8 & 1000.0 & 77.0 \\
11 & 624 & 608.0 & 185.3 & 255 & 7.2 & 14.0 & 26.5 & 29.9 & 20.7 & 993.8 & 71.0 \\
13 & 1000 & 984.0 & 299.9 & 254 & 7.5 & 14.6 & 25.6 & 30.0 & 20.1 & 981.0 & 72.0 \\
16 & 1926 & 1910.0 & 582.2 & 246 & 8.2 & 16.0 & 23.6 & 30.5 & 17.8 & 950.0 & 70.0 \\
17 & 2000 & 1984.0 & 604.7 & 245 & 8.2 & 16.0 & 23.5 & 30.6 & 17.5 & 947.7 & 69.0 \\
19 & 3000 & 2984.0 & 909.5 & 232 & 9.3 & 18.0 & 21.6 & 31.4 & 15.1 & 915.2 & 66.0 \\
20 & 3472 & 3456.0 & 1053.4 & 228 & 9.8 & 19.0 & 20.8 & 32.0 & 14.8 & 900.0 & 68.0 \\
21 & 4000 & 3984.0 & 1214.3 & 224 & 10.0 & 19.5 & 19.8 & 32.5 & 13.9 & 883.6 & 69.0 \\
22 & 5000 & 4984.0 & 1519.1 & 218 & 10.3 & 20.0 & 17.7 & 33.4 & 13.3 & 852.9 & 76.0 \\
23 & 5088 & 5072.0 & 1545.9 & 218 & 10.3 & 20.0 & 17.4 & 33.4 & 13.1 & 850.0 & 76.0 \\
24 & 5602 & 5586.0 & 1702.6 & 216 & 10.3 & 20.0 & 16.4 & 33.8 & 11.9 & 834.9 & 75.0 \\
25 & 6000 & 5984.0 & 1823.9 & 215 & 10.4 & 20.2 & 15.8 & 34.3 & 10.8 & 823.1 & 73.0 \\
26 & 6153 & 6137.0 & 1870.6 & 214 & 10.3 & 20.0 & 15.5 & 34.4 & 10.4 & 818.6 & 72.0 \\
27 & 6782 & 6766.0 & 2062.3 & 214 & 10.3 & 20.0 & 14.1 & 34.7 & 7.4 & 800.0 & 64.0 \\
28 & 7000 & 6984.0 & 2128.7 & 214 & 10.1 & 19.7 & 13.7 & 34.8 & 6.4 & 794.1 & 61.0 \\
30 & 8000 & 7984.0 & 2433.5 & 216 & 9.3 & 18.1 & 12.4 & 36.1 & -1.3 & 765.9 & 40.0 \\
31 & 8400 & 8384.0 & 2555.4 & 217 & 8.7 & 17.0 & 12.4 & 37.0 & -6.9 & 754.9 & 25.0 \\
32 & 8561 & 8545.0 & 2604.5 & 217 & 8.7 & 17.0 & 12.1 & 37.3 & -8.0 & 750.0 & 24.0 \\
33 & 9000 & 8984.0 & 2738.3 & 218 & 8.2 & 15.9 & 11.3 & 37.7 & -10.5 & 738.6 & 21.0
\end{bmatrix}
$$

$$
d2 := \begin{bmatrix}
4 & 1000 & 984.0 & 299.9 & 224 & 6.2 & 12.0 & 28.7 & 28.9 & 21.6 & 977.7 & 66.0 \\
7 & 2000 & 1984.0 & 604.7 & 232 & 6.4 & 12.4 & 26.6 & 27.0 & 19.6 & 944.8 & 65.0 \\
10 & 3000 & 2984.0 & 909.5 & 238 & 6.1 & 11.8 & 23.7 & 24.3 & 18.3 & 912.8 & 72.0 \\
13 & 4000 & 3984.0 & 1214.3 & 248 & 5.7 & 11.0 & 20.6 & 21.4 & 17.9 & 881.5 & 84.0 \\
16 & 5000 & 4984.0 & 1519.1 & 261 & 5.4 & 10.5 & 18.1 & 19.0 & 15.9 & 851.0 & 87.0 \\
19 & 6000 & 5984.0 & 1823.9 & 275 & 5.2 & 10.1 & 16.6 & 17.6 & 11.5 & 821.3 & 72.0 \\
22 & 7000 & 6984.0 & 2128.7 & 285 & 4.8 & 9.3 & 15.6 & 16.7 & 4.5 & 792.5 & 48.0 \\
23 & 8000 & 7984.0 & 2433.5 & 289 & 3.6 & 7.0 & 14.3 & 15.4 & -0.5 & 764.5 & 36.0 \\
24 & 9000 & 8984.0 & 2738.3 & 302 & 1.7 & 3.3 & 13.1 & 14.3 & -4.8 & 737.4 & 28.0 \\
25 & 10000 & 9984.0 & 3043.1 & 1+360 & 1.0 & 2.0 & 11.5 & 12.7 & -3.8 & 711.1 & 35.0 \\
26 & 11000 & 10984.0 & 3347.9 & 20+360 & 1.3 & 2.5 & 9.1 & 10.1 & 1.0 & 685.7 & 57.0 \\
27 & 12000 & 11984.0 & 3652.7 & 20+360 & 1.3 & 2.5 & 7.3 & 8.2 & -5.3 & 660.9 & 41.0
\end{bmatrix}
$$

$$
d21 := \begin{bmatrix}
5 & 212 & 196.0 & 59.7 & 69 & 3.6 & 7.0 & 22.8 & 24.0 & 18.7 & 1014.8 & 78.0 \\
9 & 631 & 615.0 & 187.5 & 73 & 3.6 & 7.0 & 21.6 & 23.7 & 16.3 & 1000.0 & 72.0 \\
11 & 1000 & 984.0 & 299.9 & 74 & 3.7 & 7.2 & 20.6 & 23.7 & 15.6 & 987.2 & 73.0 \\
14 & 2000 & 1984.0 & 604.7 & 82 & 3.0 & 5.8 & 17.4 & 23.4 & 14.5 & 652.9 & 83.0 \\
15 & 2083 & 2067.0 & 630.0 & 84 & 3.1 & 6.0 & 17.1 & 23.4 & 14.5 & 950.0 & 85.0 \\
16 & 2261 & 2245.0 & 684.3 & 87 & 2.6 & 5.0 & 16.5 & 23.3 & 14.5 & 944.1 & 88.0 \\
17 & 2832 & 2816.0 & 858.3 & 106 & 2.1 & 4.0 & 15.0 & 23.5 & 14.3 & 925.0 & 96.0 \\
18 & 3000 & 2984.0 & 909.5 & 115 & 1.9 & 3.6 & 14.4 & 23.3 & 13.7 & 919.5 & 95.0 \\
19 & 3219 & 3203.0 & 976.3 & 131 & 1.5 & 3.0 & 13.7 & 23.2 & 12.9 & 912.3 & 95.0 \\
20 & 3589 & 3573.0 & 1089.1 & 149 & 1.0 & 2.0 & 12.9 & 23.5 & 12.0 & 900.0 & 95.0 \\
21 & 3786 & 3770.0 & 1149.1 & 166 & 1.0 & 2.0 & 12.5 & 23.6 & 11.6 & 893.8 & 94.0 \\
24 & 4000 & 3984.0 & 1214.3 & 225 & 1.0 & 1.9 & 12.0 & 23.7 & 11.2 & 886.9 & 95.0 \\
27 & 4907 & 4891.0 & 1490.8 & 304 & 3.6 & 7.0 & 11.4 & 25.2 & 2.1 & 858.2 & 53.0 \\
28 & 5161 & 5145.0 & 1568.2 & 303 & 5.1 & 10.0 & 11.7 & 26.1 & -2.4 & 850.0 & 39.0 \\
29 & 6000 & 5984.0 & 1823.9 & 298 & 8.4 & 16.3 & 11.0 & 27.7 & -7.1 & 824.7 & 27.0 \\
30 & 6819 & 6803.0 & 2073.6 & 290 & 10.3 & 20.0 & 9.7 & 28.9 & -9.2 & 800.0 & 26.0 \\
31 & 7195 & 7179.0 & 2188.2 & 287 & 10.8 & 21.0 & 9.4 & 29.7 & -11.8 & 789.4 & 21.0 \\
32 & 8000 & 7984.0 & 2433.5 & 284 & 11.9 & 23.2 & 7.5 & 30.9 & 1.7 & 766.3 & 67.0 \\
33 & 8569 & 8553.0 & 2607 & 283 & 12.3 & 24.0 & 7.0 & 32.4 & 3.5 & 750.0 & 78.0
\end{bmatrix}
$$

$$d23 := \begin{bmatrix} 6 & 250 & 234.0 & 71.3 & 257 & 3.1 & 6.0 & 25.5 & 27.9 & 22.7 & 1008.3 & 84.0 \\ 9 & 488 & 472.0 & 143.9 & 246 & 3.6 & 7.0 & 25.3 & 28.4 & 22.5 & 1000.0 & 85.0 \\ 12 & 1000 & 984.0 & 299.9 & 233 & 4.1 & 8.0 & 25.5 & 30.1 & 21.9 & 982.5 & 81.0 \\ 13 & 1142 & 1126.0 & 343.2 & 231 & 4.1 & 8.0 & 25.7 & 30.7 & 21.6 & 977.8 & 78.0 \\ 15 & 1686 & 1670.0 & 509.0 & 227 & 4.6 & 9.0 & 25.4 & 31.8 & 20.1 & 959.6 & 72.0 \\ 16 & 2000 & 1684.0 & 604.7 & 225 & 5.0 & 9.7 & 24.8 & 32.1 & 19.8 & 949.3 & 74.0 \\ 17 & 2606 & 2590.0 & 789.4 & 225 & 5.1 & 10.0 & 23.7 & 32.7 & 18.9 & 929.6 & 75.0 \\ 18 & 3000 & 2984.0 & 909.5 & 225 & 5.0 & 9.7 & 22.6 & 32.8 & 18.4 & 916.9 & 77.0 \\ 19 & 3528 & 3512.0 & 1070.5 & 223 & 4.6 & 9.0 & 21.4 & 33.1 & 17.9 & 900.0 & 81.0 \\ 20 & 4000 & 3984.0 & 1214.3 & 223 & 4.0 & 7.8 & 20.1 & 33.2 & 17.5 & 885.4 & 85.0 \\ 21 & 4717 & 4701.0 & 1432.9 & 226 & 3.1 & 6.0 & 18.2 & 33.3 & 16.5 & 863.3 & 90.0 \\ 22 & 5000 & 4984.0 & 1519.1 & 230 & 2.5 & 4.8 & 17.9 & 33.7 & 15.3 & 854.7 & 85.0 \\ 23 & 5147 & 5131.0 & 1563.9 & 233 & 2.1 & 4.0 & 17.7 & 33.9 & 14.7 & 850.0 & 83.0 \\ 25 & 6000 & 5984.0 & 1823.9 & 256 & 1.2 & 2.4 & 16.3 & 34.7 & 11.7 & 824.9 & 74.0 \\ 26 & 6323 & 6307.0 & 1922.4 & 269 & 1.0 & 2.0 & 15.5 & 34.8 & 10.8 & 815.4 & 74.0 \\ 29 & 6844 & 6828.0 & 2081.2 & 328 & 1.0 & 1.9 & 14.9 & 35.5 & 7.1 & 800.0 & 59.0 \\ 31 & 7000 & 6984.0 & 2128.7 & 357 & 1.0 & 1.9 & 14.7 & 35.6 & 5.3 & 795.9 & 54.0 \\ 32 & 7427 & 7411.0 & 2258.9 & 11+360 & 1.0 & 1.9 & 14.2 & 36.0 & -0.3 & 783.8 & 37.0 \\ 33 & 8628 & 8612.0 & 2624.9 & 22+360 & 1.0 & 1.9 & 11.9 & 37.6 & 1.6 & 750.0 & 49.0 \end{bmatrix}$$

$i16 := 1 .. rows(d16)$ $\quad i19 := 1 .. rows(d19)$ $\quad i2 := 1 .. rows(d2)$ $\quad i21 := 1 .. rows(d21)$ $\quad i23 := 1 .. rows(d23)$

$i16 := 1 .. rows(d16)$

Renaming data elements:

$ML16_{i16} := d16_{i16,1}$ $\quad dt16_{i16} := d16_{i16,10}$ $\quad rh16_{i16} := d16_{i16,12}$

$alt16_{i16} := d16_{i16,4}$ $\quad t16_{i16} := d16_{i16,8}$

$wd16_{i16} := d16_{i16,5}$ $\quad pt16_{i16} := d16_{i16,9}$

$ws16_{i16} := d16_{i16,6}$ $\quad p16_{i16} := d16_{i16,11}$

$i19 := 1 .. rows(d19)$ $\quad$ $i2 := 1 .. rows(d2)$ $\quad$ $i21 := 1 .. rows(d21)$ $\quad$ $i23 := 1 .. rows(d23)$

$ML19_{i19} := d19_{i19,1}$ $\quad$ $ML2_{i2} := d2_{i2,1}$ $\quad$ $ML21_{i21} := d21_{i21,1}$ $\quad$ $ML23_{i23} := d23_{i23,1}$

$alt19_{i19} := d19_{i19,4}$ $\quad$ $alt2_{i2} := d2_{i2,4}$ $\quad$ $alt21_{i21} := d21_{i21,4}$ $\quad$ $alt23_{i23} := d23_{i23,4}$

$wd19_{i19} := d19_{i19,5}$ $\quad$ $wd2_{i2} := d2_{i2,5}$ $\quad$ $wd21_{i21} := d21_{i21,5}$ $\quad$ $wd23_{i23} := d23_{i23,5}$

$ws19_{i19} := d19_{i19,6}$ $\quad$ $ws2_{i2} := d2_{i2,6}$ $\quad$ $ws21_{i21} := d21_{i21,6}$ $\quad$ $ws23_{i23} := d23_{i23,6}$

$t19_{i19} := d19_{i19,8}$ $\quad$ $t2_{i2} := d2_{i2,8}$ $\quad$ $t21_{i21} := d21_{i21,8}$ $\quad$ $t23_{i23} := d23_{i23,8}$

$pt19_{i19} := d19_{i19,9}$ $\quad$ $pt2_{i2} := d2_{i2,9}$ $\quad$ $pt21_{i21} := d21_{i21,9}$ $\quad$ $pt23_{i23} := d23_{i23,9}$

$p19_{i19} := d19_{i19,11}$ $\quad$ $p2_{i2} := d2_{i2,11}$ $\quad$ $p21_{i21} := d21_{i21,11}$ $\quad$ $p23_{i23} := d23_{i23,11}$

$rh19_{i19} := d19_{i19,12}$ $\quad$ $rh2_{i2} := d2_{i2,12}$ $\quad$ $rh21_{i21} := d21_{i21,12}$ $\quad$ $rh23_{i23} := d23_{i23,12}$

$dt19_{i19} := d19_{i19,10}$ $\quad$ $dt2_{i2} := d2_{i2,10}$ $\quad$ $dt21_{i21} := d21_{i21,10}$ $\quad$ $dt23_{i23} := d23_{i23,10}$

Potential Temperature Lapse Rate Calculations:

$j16 := 1 .. rows(d16) - 1$ $\quad$ $j19 := 1 .. rows(d19) - 1$ $\quad$ $j2 := 1 .. rows(d2) - 1$

$$d\Theta dz16_{j16} := \frac{pt16_{j16+1} - pt16_{j16}}{alt16_{j16+1} - alt16_{j16}} \qquad d\Theta dz19_{j19} := \frac{pt19_{j19+1} - pt19_{j19}}{alt19_{j19+1} - alt19_{j19}} \qquad d\Theta dz2_{j2} := \frac{pt2_{j2+1} - pt2_{j2}}{alt2_{j2+1} - alt2_{j2}}$$

$j21 := 1 .. rows(d21) - 1$ $\quad$ $j23 := 1 .. rows(d23) - 1$

$$d\Theta dz21_{j21} := \frac{pt21_{j21+1} - pt21_{j21}}{alt21_{j21+1} - alt21_{j21}} \qquad d\Theta dz23_{j23} := \frac{pt23_{j23+1} - pt23_{j23}}{alt23_{j23+1} - alt23_{j23}}$$

$\gamma 2 := d\Theta dz2$ $\quad$ $\gamma 16 := d\Theta dz16$ $\quad$ $\gamma 19 := d\Theta dz19$ $\quad$ $\gamma 21 := d\Theta dz21$ $\quad$ $\gamma 23 := d\Theta dz23$

Potential Temperature Lapse Rate Mean and Variance Calculations:

$m\_\gamma 2 := mean(\gamma 2)$ $\quad$ $m\_\gamma 2 = -6.174 \cdot 10^{-3}$ $\quad$ $v\_\gamma 2 := var(\gamma 2)$ $\quad$ $v\_\gamma 2 = 4.615 \cdot 10^{-6}$

$m\_\gamma 16 := mean(\gamma 16)$ $\quad$ $m\_\gamma 16 = 4.635 \cdot 10^{-3}$ $\quad$ $v\_\gamma 16 := var(\gamma 16)$ $\quad$ $v\_\gamma 16 = 8.842 \cdot 10^{-6}$

$m\_\gamma 19 := mean(\gamma 19)$ $\quad$ $m\_\gamma 19 = 3.974 \cdot 10^{-3}$ $\quad$ $v\_\gamma 19 := var(\gamma 19)$ $\quad$ $v\_\gamma 19 = 9.654 \cdot 10^{-6}$

$m\_\gamma 21 := mean(\gamma 21)$ $\quad$ $m\_\gamma 21 = 2.504 \cdot 10^{-3}$ $\quad$ $v\_\gamma 21 := var(\gamma 21)$ $\quad$ $v\_\gamma 21 = 1.68 \cdot 10^{-5}$

$m\_\gamma 23 := mean(\gamma 23)$ $\quad$ $m\_\gamma 23 = 4.203 \cdot 10^{-3}$ $\quad$ $v\_\gamma 23 := var(\gamma 23)$ $\quad$ $v\_\gamma 23 = 1.188 \cdot 10^{-5}$

$v\_t\gamma := (v\_\gamma 2 \;\; v\_\gamma 16 \;\; v\_\gamma 19 \;\; v\_\gamma 21 \;\; v\_\gamma 23)$

$m\_t\gamma := (m\_\gamma 2 \;\; m\_\gamma 16 \;\; m\_\gamma 19 \;\; m\_\gamma 21 \;\; m\_\gamma 23)$ $\quad$ $m\_\gamma := m\_t\gamma^T$ $\quad$ $v\_\gamma := v\_t\gamma^T$

$v\_t\gamma = (4.615 \cdot 10^{-6} \;\; 8.842 \cdot 10^{-6} \;\; 9.654 \cdot 10^{-6} \;\; 1.68 \cdot 10^{-5} \;\; 1.188 \cdot 10^{-5})$

$m\_t\gamma = (-6.174 \cdot 10^{-3} \;\; 4.635 \cdot 10^{-3} \;\; 3.974 \cdot 10^{-3} \;\; 2.504 \cdot 10^{-3} \;\; 4.203 \cdot 10^{-3})$

Potential Temperature Mean and Variance Calculations:

$m\_pt2 := mean(pt2)$ $m\_pt2 = 17.967$ $v\_pt2 := var(pt2)$ $v\_pt2 = 38.407$

$m\_pt16 := mean(pt16)$ $m\_pt16 = 24.479$ $v\_pt16 := var(pt16)$ $v\_pt16 = 9.556$

$m\_pt19 := mean(pt19)$ $m\_pt19 = 33.075$ $v\_pt19 := var(pt19)$ $v\_pt19 = 7.297$

$m\_pt21 := mean(pt21)$ $m\_pt21 = 25.432$ $v\_pt21 := var(pt21)$ $v\_pt21 = 8.344$

$m\_pt23 := mean(pt23)$ $m\_pt23 = 33.047$ $v\_pt23 := var(pt23)$ $v\_pt23 = 5.997$

$v\_tpt := (v\_pt2\ \ v\_pt16\ \ v\_pt19\ \ v\_pt21\ \ v\_pt23)$

$m\_tpt := (m\_pt2\ \ m\_pt16\ \ m\_pt19\ \ m\_pt21\ \ m\_pt23)$ $m\_pt := m\_tpt^T$ $v\_pt := v\_tpt^T$

$v\_tpt = (\ 38.407\ \ 9.556\ \ 7.297\ \ 8.344\ \ 5.997\ )$

$m\_tpt = (\ 17.967\ \ 24.479\ \ 33.075\ \ 25.432\ \ 33.047\ )$

Wind Speed Mean and Variance Calculations:

$m\_ws2 := mean(ws2)$ $m\_ws2 = 4.058$ $v\_ws2 := var(ws2)$ $v\_ws2 = 4.244$

$m\_ws16 := mean(ws16)$ $m\_ws16 = 6.729$ $v\_ws16 := var(ws16)$ $v\_ws16 = 0.308$

$m\_ws19 := mean(ws19)$ $m\_ws19 = 9.025$ $v\_ws19 := var(ws19)$ $v\_ws19 = 1.58$

$m\_ws21 := mean(ws21)$ $m\_ws21 = 4.763$ $v\_ws21 := var(ws21)$ $v\_ws21 = 14.334$

$m\_ws23 := mean(ws23)$ $m\_ws23 = 3.005$ $v\_ws23 := var(ws23)$ $v\_ws23 = 2.402$

$v\_tws := (v\_ws2\ \ v\_ws16\ \ v\_ws19\ \ v\_ws21\ \ v\_ws23)$ $v\_ws := v\_tws^T$

$m\_tws := (m\_ws2\ \ m\_ws16\ \ m\_ws19\ \ m\_ws21\ \ m\_ws23)$ $m\_ws := m\_tws^T$

$v\_tws = (\ 4.244\ \ 0.308\ \ 1.58\ \ 14.334\ \ 2.402\ )$ $m\_tws = (\ 4.058\ \ 6.729\ \ 9.025\ \ 4.763\ \ 3.005\ )$

Wind Direction Mean and Variance Calculations:

$m\_wd2 := mean(wd2)$ $m\_wd2 = 289.583$ $v\_wd2 := var(wd2)$ $v\_wd2 = 2.889 \cdot 10^3$

$m\_wd16 := mean(wd16)$ $m\_wd16 = 330.214$ $v\_wd16 := var(wd16)$ $v\_wd16 = 1.29 \cdot 10^3$

$m\_wd19 := mean(wd19)$ $m\_wd19 = 228.55$ $v\_wd19 := var(wd19)$ $v\_wd19 = 254.448$

$m\_wd21 := mean(wd21)$ $m\_wd21 = 179.474$ $v\_wd21 := var(wd21)$ $v\_wd21 = 8.798 \cdot 10^3$

$m\_wd23 := mean(wd23)$ $m\_wd23 = 261.421$ $v\_wd23 := var(wd23)$ $v\_wd23 = 2.808 \cdot 10^3$

$v\_twd := (v\_wd2\ \ v\_wd16\ \ v\_wd19\ \ v\_wd21\ \ v\_wd23)$ $v\_wd := v\_twd^T$

$m\_twd := (m\_wd2\ \ m\_wd16\ \ m\_wd19\ \ m\_wd21\ \ m\_wd23)$ $m\_wd := m\_twd^T$

$v\_twd = (\ 2889\ \ 1290\ \ 254\ \ 8798\ \ 2808\ )$ $m\_twd = (\ 289.6\ \ 330.2\ \ 228.6\ \ 179.5\ \ 261.4\ )$

Relative Humidity Mean and Variance Calculations:

$m\_rh2 := mean(rh2)$	$m\_rh2 = 57.583$	$v\_rh2 := var(rh2)$	$v\_rh2 = 360.243$
$m\_rh16 := mean(rh16)$	$m\_rh16 = 88.143$	$v\_rh16 := var(rh16)$	$v\_rh16 = 25.98$
$m\_rh19 := mean(rh19)$	$m\_rh19 = 62.7$	$v\_rh19 := var(rh19)$	$v\_rh19 = 345.21$
$m\_rh21 := mean(rh21)$	$m\_rh21 = 71.579$	$v\_rh21 := var(rh21)$	$v\_rh21 = 632.033$
$m\_rh23 := mean(rh23)$	$m\_rh23 = 73.526$	$v\_rh23 := var(rh23)$	$v\_rh23 = 186.46$

$v\_trh := (v\_rh2 \;\; v\_rh16 \;\; v\_rh19 \;\; v\_rh21 \;\; v\_rh23)$

$m\_trh := (m\_rh2 \;\; m\_rh16 \;\; m\_rh19 \;\; m\_rh21 \;\; m\_rh23)$ $\quad m\_rh := m\_trh^T \quad v\_rh := v\_trh^T$

$v\_trh = (360 \;\; 26 \;\; 345 \;\; 632 \;\; 186)$ $\quad m\_trh = (57.6 \;\; 88.1 \;\; 62.7 \;\; 71.6 \;\; 73.5)$ $\quad s := 1..5$

Temperature Mean and Variance Calculations:

$m\_t2 := mean(t2)$	$m\_t2 = 17.1$	$v\_t2 := var(t2)$	$v\_t2 = 41.613$
$m\_t16 := mean(t16)$	$m\_t16 = 15.771$	$v\_t16 := var(t16)$	$v\_t16 = 5.079$
$m\_t19 := mean(t19)$	$m\_t19 = 18.615$	$v\_t19 := var(t19)$	$v\_t19 = 26.398$
$m\_t21 := mean(t21)$	$m\_t21 = 13.905$	$v\_t21 := var(t21)$	$v\_t21 = 19.298$
$m\_t23 := mean(t23)$	$m\_t23 = 20.068$	$v\_t23 := var(t23)$	$v\_t23 = 20.834$

$v\_tt := (v\_t2 \;\; v\_t16 \;\; v\_t19 \;\; v\_t21 \;\; v\_t23)$

$m\_tt := (m\_t2 \;\; m\_t16 \;\; m\_t19 \;\; m\_t21 \;\; m\_t23)$ $\quad m\_t := m\_tt^T \quad v\_t := v\_tt^T$

$v\_tt = (42 \;\; 5 \;\; 26 \;\; 19 \;\; 21)$ $\quad m\_tt = (17.1 \;\; 15.8 \;\; 18.6 \;\; 13.9 \;\; 20.1)$

Dew Point Temperature Mean and Variance Calculations:

$m\_dt2 := mean(dt2)$	$m\_dt2 = 7.992$	$v\_dt2 := var(dt2)$	$v\_dt2 = 100.979$
$m\_dt16 := mean(dt16)$	$m\_dt16 = 13.8$	$v\_dt16 := var(dt16)$	$v\_dt16 = 4.261$
$m\_dt19 := mean(dt19)$	$m\_dt19 = 10.57$	$v\_dt19 := var(dt19)$	$v\_dt19 = 95.219$
$m\_dt21 := mean(dt21)$	$m\_dt21 = 7.716$	$v\_dt21 := var(dt21)$	$v\_dt21 = 85.292$
$m\_dt23 := mean(dt23)$	$m\_dt23 = 14.947$	$v\_dt23 := var(dt23)$	$v\_dt23 = 47.276$

$v\_tdt := (v\_dt2 \;\; v\_dt16 \;\; v\_dt19 \;\; v\_dt21 \;\; v\_dt23)$

$m\_tdt := (m\_dt2 \;\; m\_dt16 \;\; m\_dt19 \;\; m\_dt21 \;\; m\_dt23)$ $\quad m\_dt := m\_tdt^T \quad v\_dt := v\_tdt^T$

$v\_tdt = (101 \;\; 4 \;\; 95 \;\; 85 \;\; 47)$ $\quad m\_tdt = (8 \;\; 13.8 \;\; 10.6 \;\; 7.7 \;\; 14.9)$

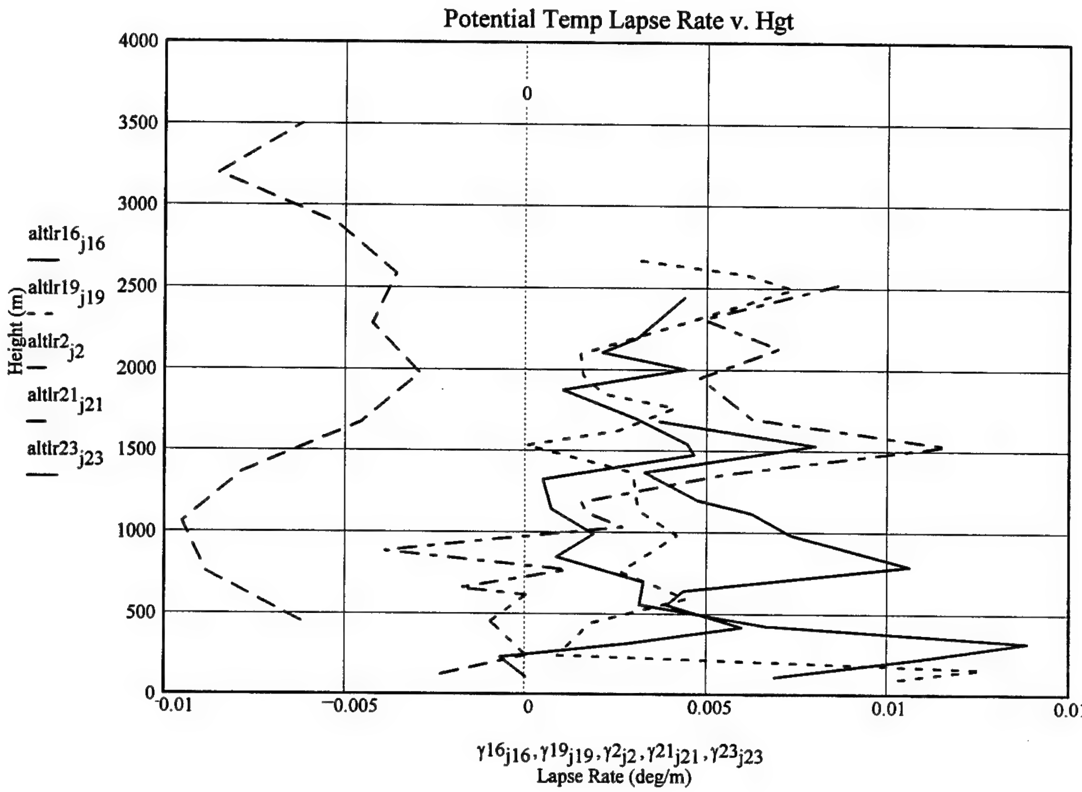
$zt := (0 \;\; 0 \;\; 0 \;\; 0 \;\; 0) \quad z := zt^T$

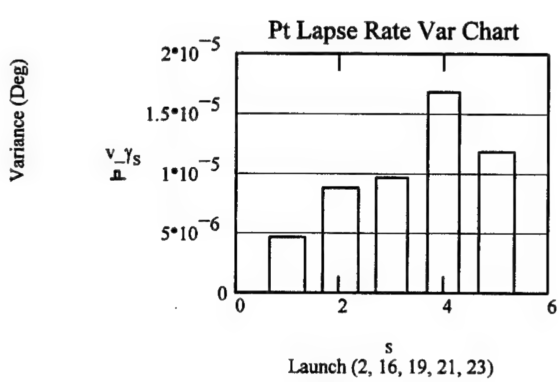
$$altlr2_{j2} := alt2_{j2} + \left(\frac{alt2_{j2+1} - alt2_{j2}}{2}\right) \quad altlr16_{j16} := alt16_{j16} + \left(\frac{alt16_{j16+1} - alt16_{j16}}{2}\right)$$

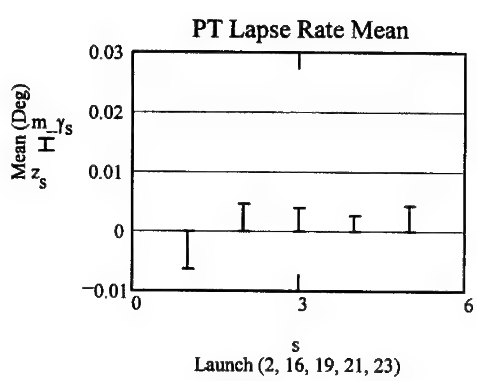
$$altlr21_{j21} := alt21_{j21} + \left(\frac{alt21_{j21+1} - alt21_{j21}}{2}\right) \quad altlr23_{j23} := alt23_{j23} + \left(\frac{alt23_{j23+1} - alt23_{j23}}{2}\right)$$

$$altlr19_{j19} := alt19_{j19} + \left(\frac{alt19_{j19+1} - alt19_{j19}}{2}\right)$$

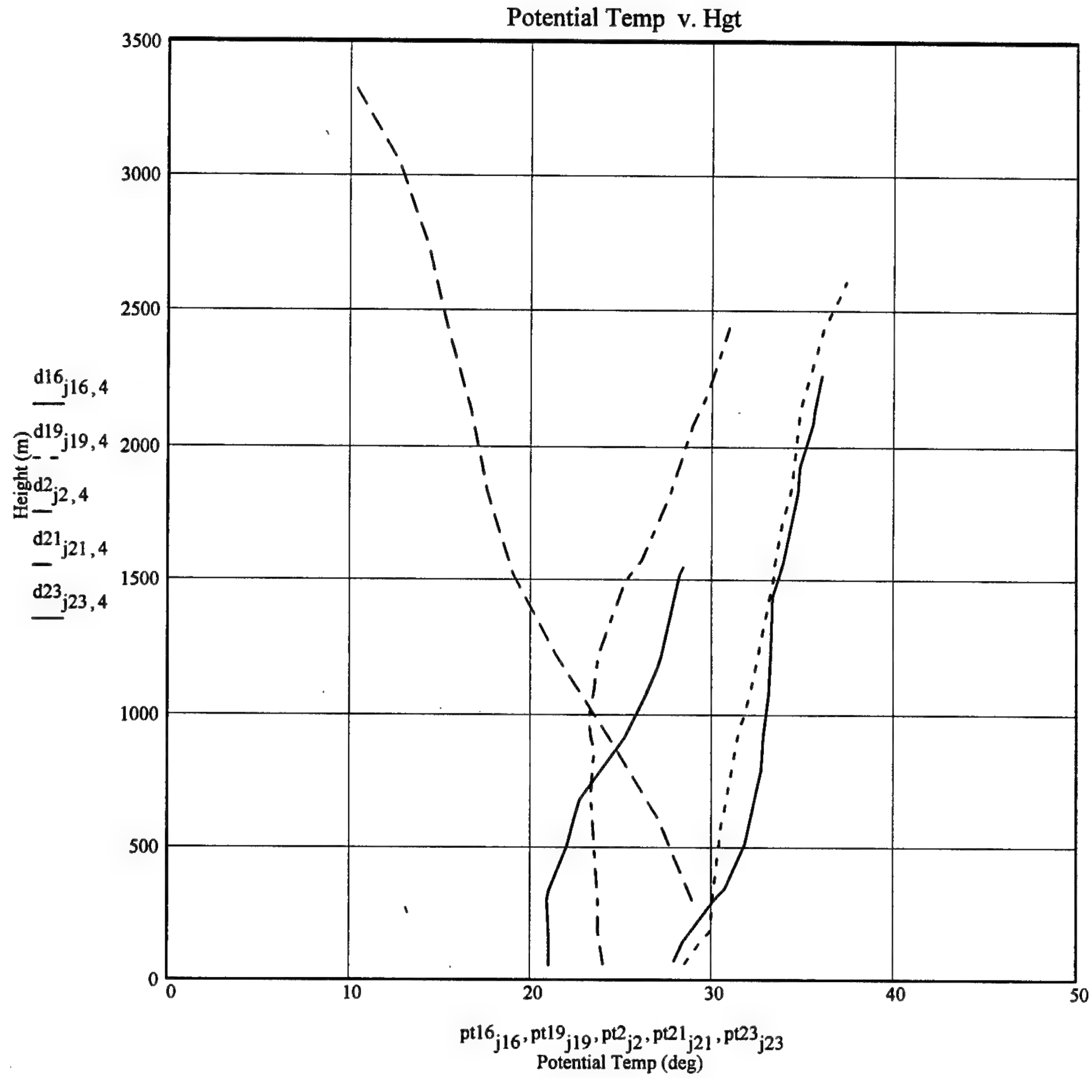
Potential Temperature Lapse Rate Profiles:

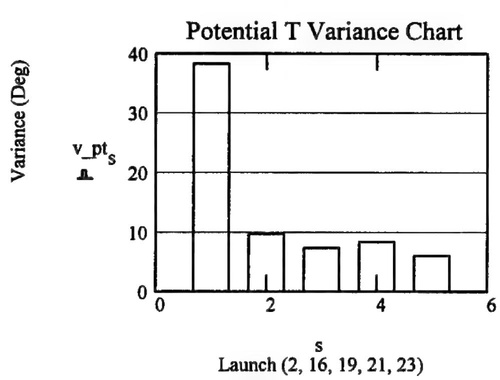


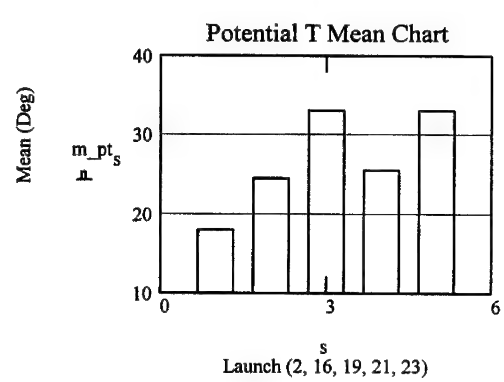




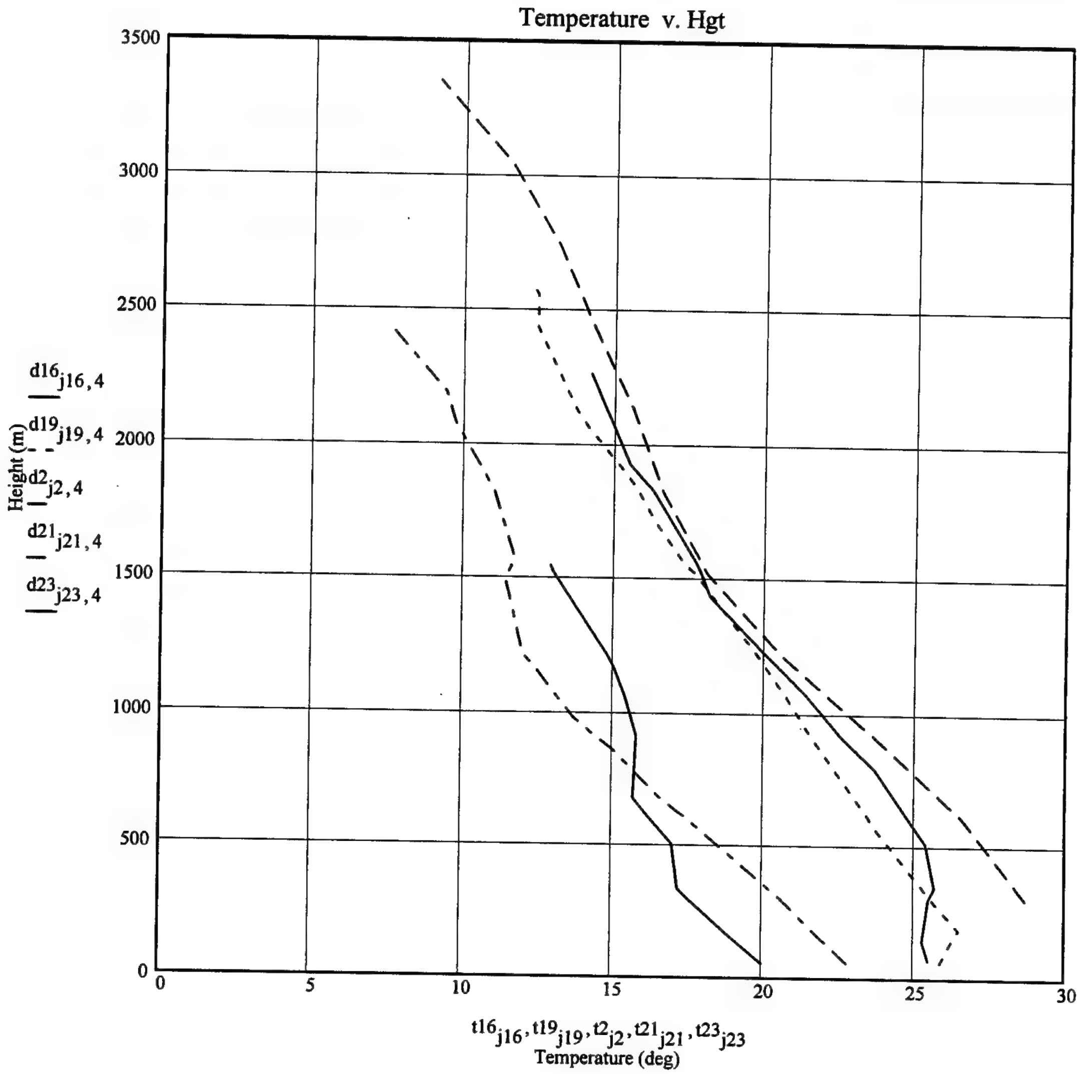
Potential Temperature Profiles:

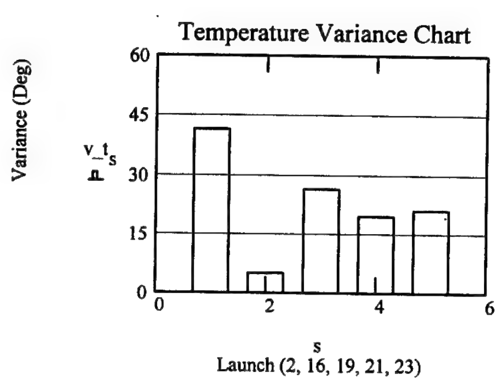


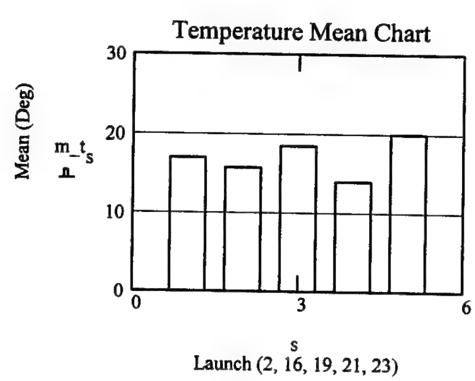




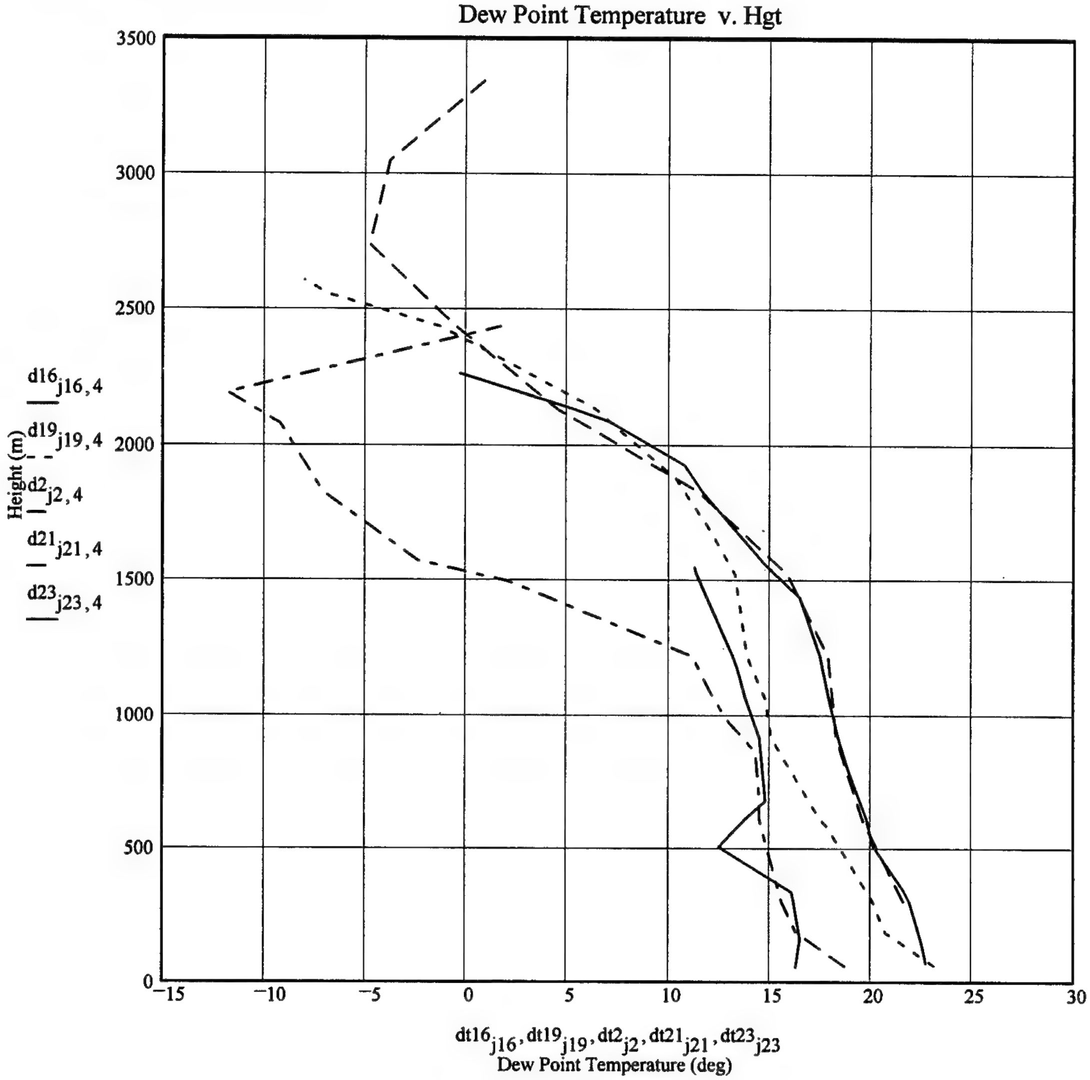
Temperature Profiles:

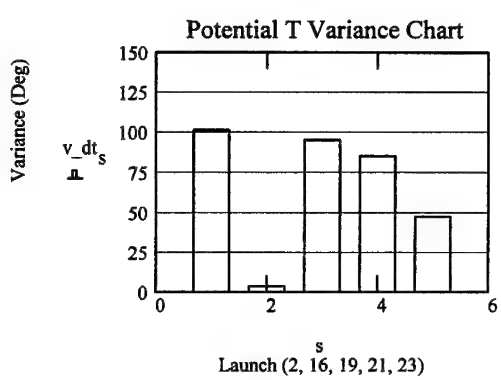


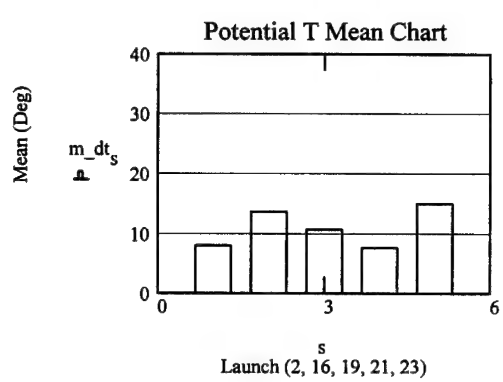




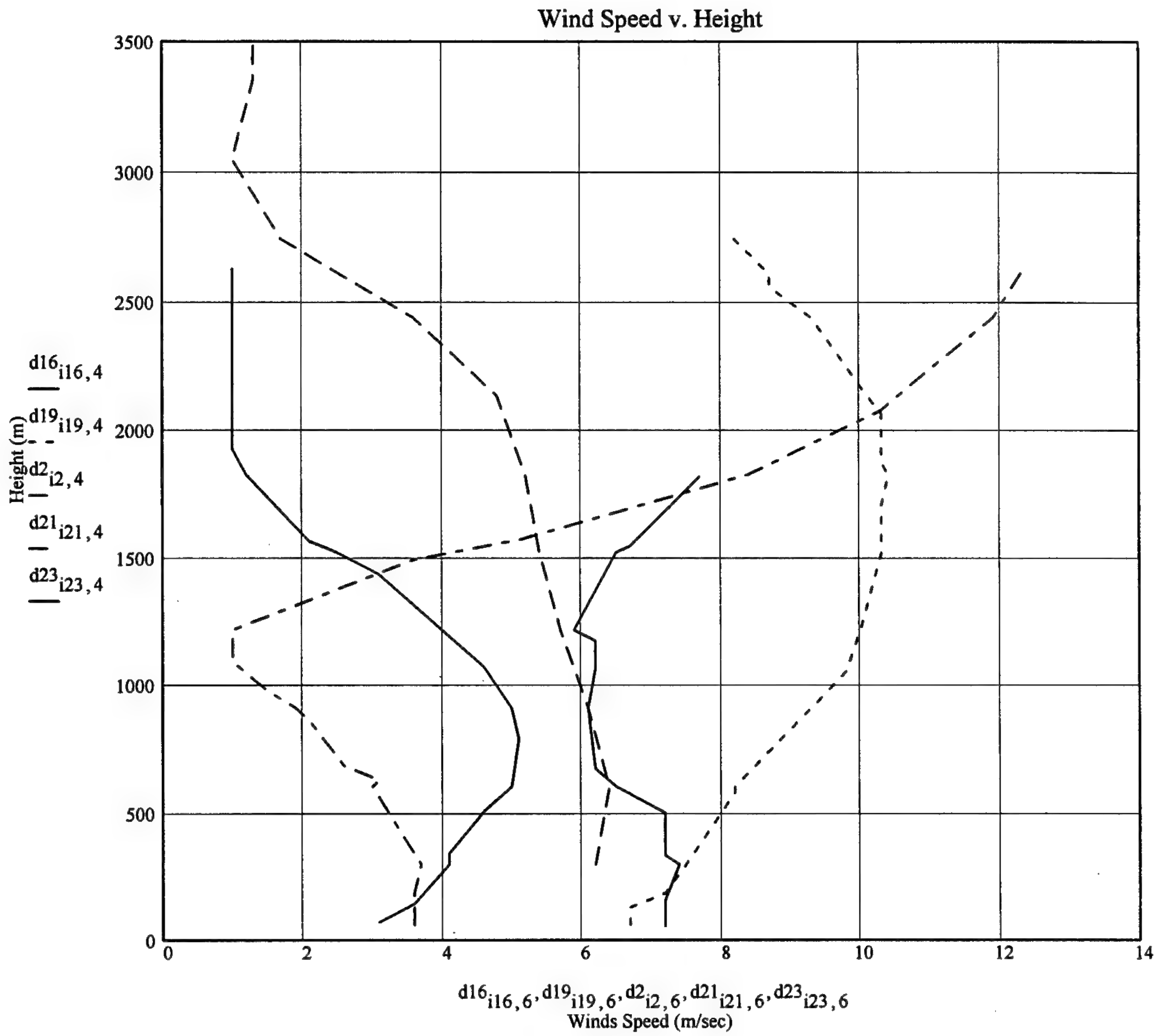
Dewpoint Point Temperature Profiles:

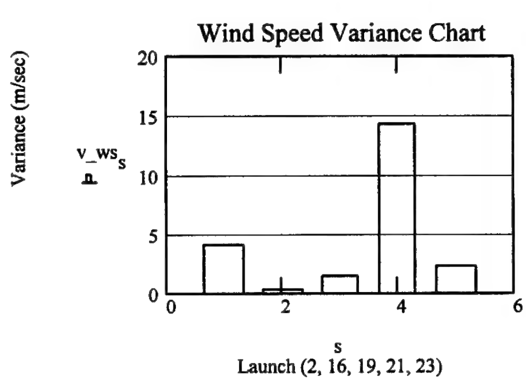


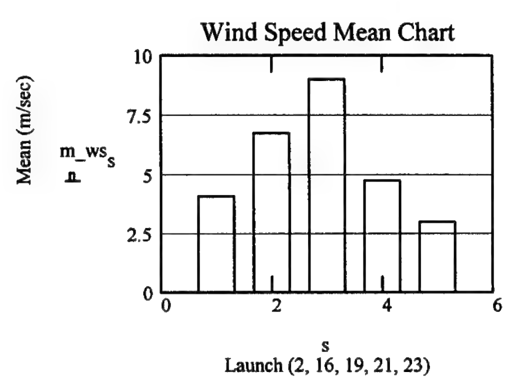




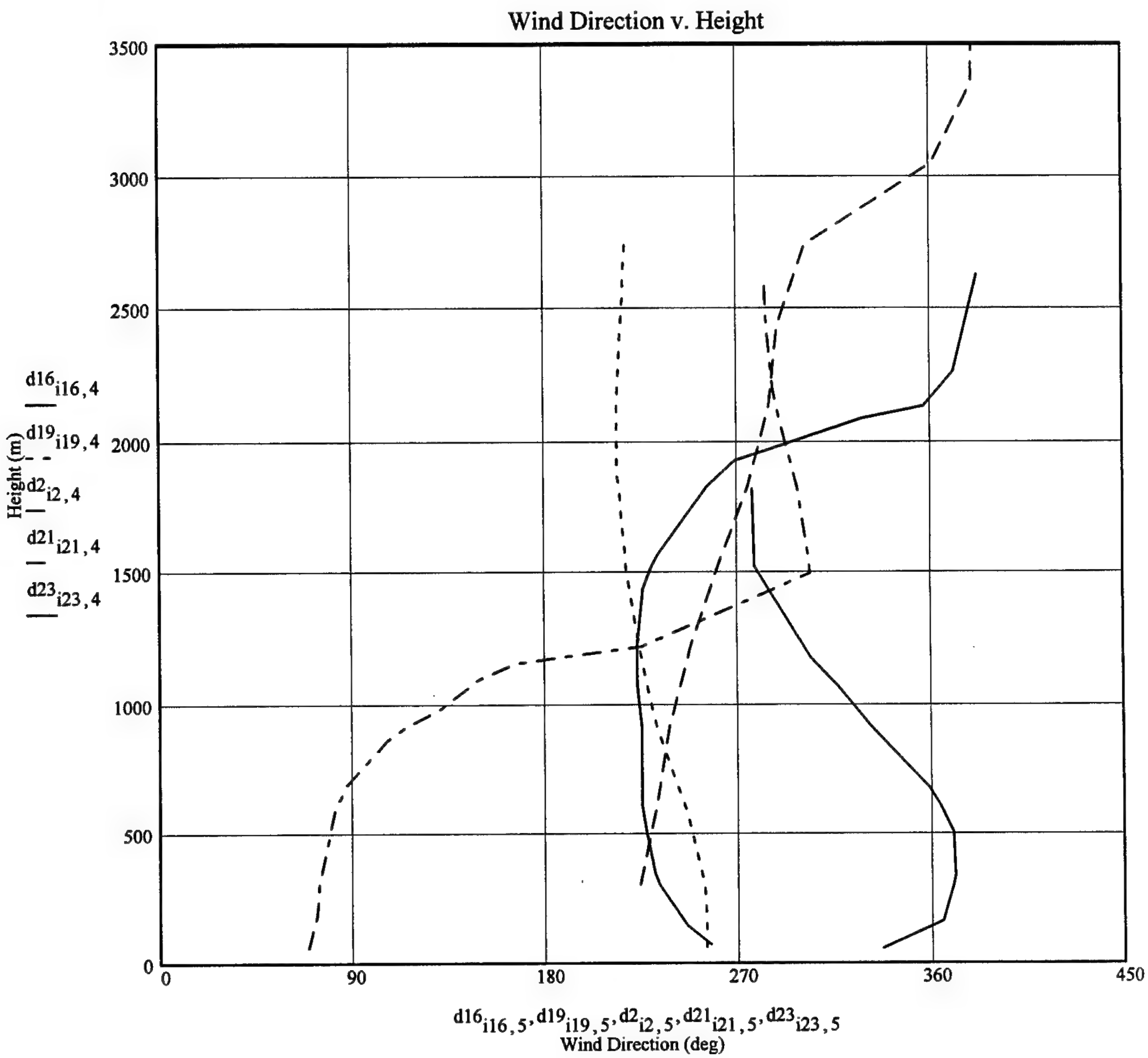
Wind Speed Profiles:

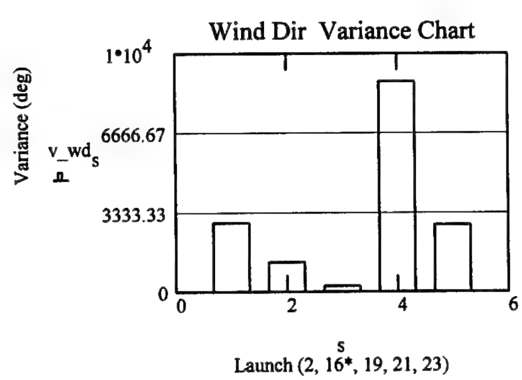


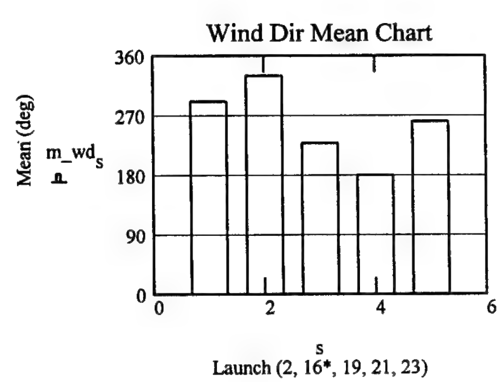




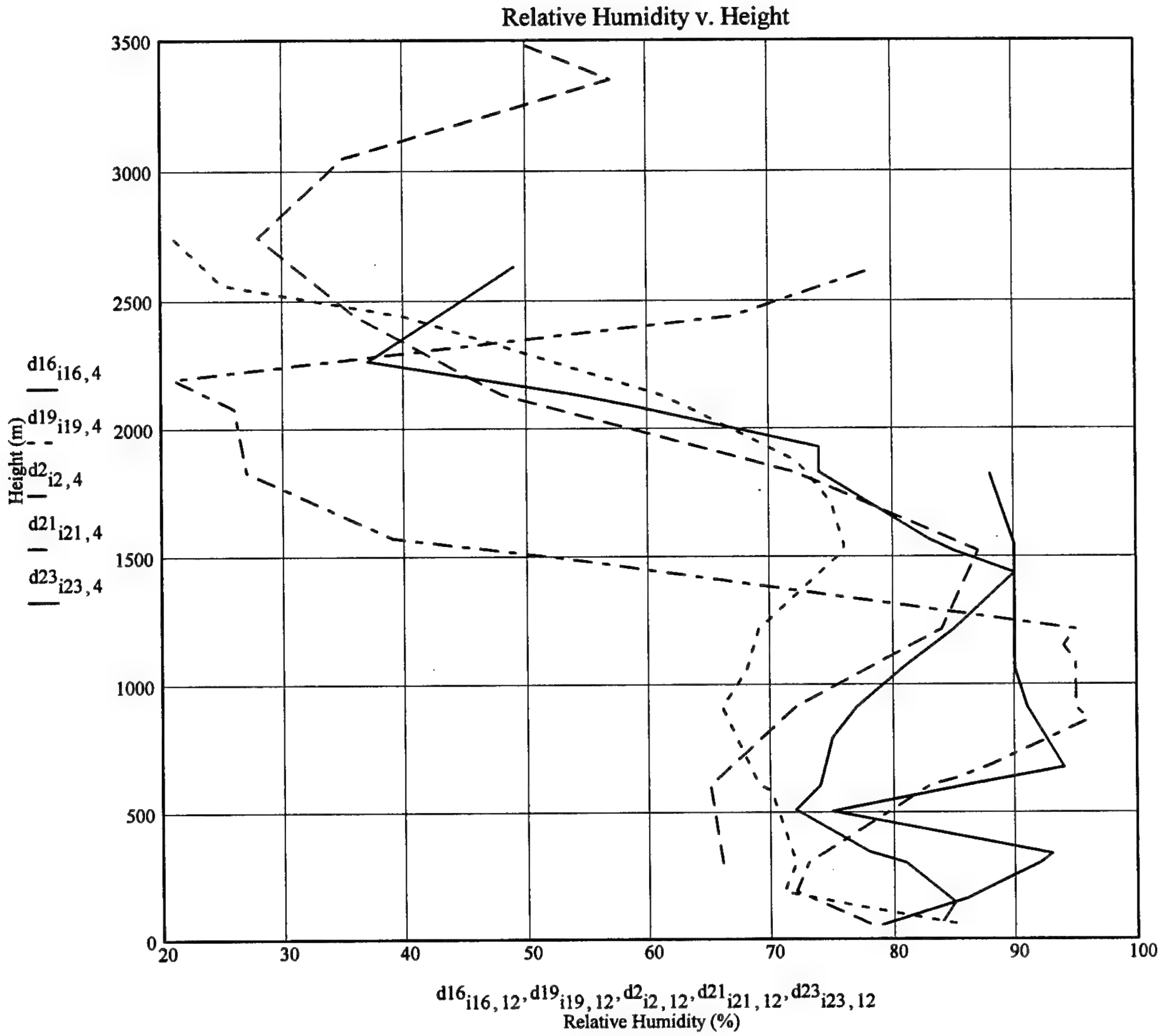
Wind Direction Profiles:

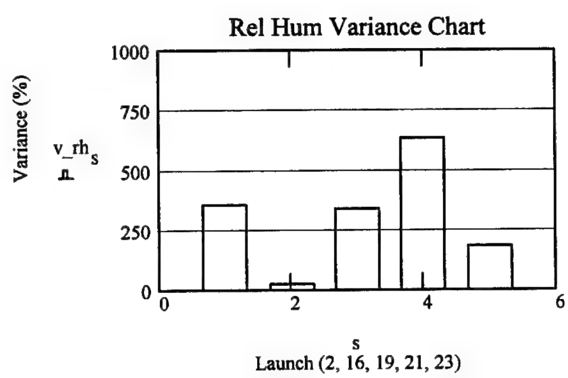


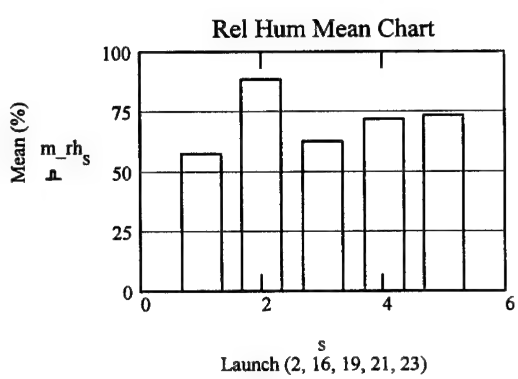




Relative Humidity Profiles:







Data for first 2000 meters:

ORIGIN ≡ 1

$$d16 := \begin{bmatrix}
12 & 1000 & 984.0 & 299.9 & 10+180 & 7.4 & 14.4 & 17.5 & 20.9 & 16.2 & 983.9 & 92.0 \\
13 & 1114 & 1098.0 & 334.7 & 11+180 & 7.2 & 14.0 & 17.2 & 21.0 & 16.1 & 979.9 & 93.0 \\
15 & 1663 & 1647.0 & 502.0 & 10+180 & 7.2 & 14.0 & 17.0 & 22.0 & 12.5 & 961.0 & 75.0 \\
16 & 2000 & 1984.0 & 604.7 & 4+180 & 6.5 & 12.6 & 16.2 & 22.4 & 13.8 & 949.5 & 86.0 \\
17 & 2226 & 2210.0 & 673.6 & 359 & 6.2 & 12.0 & 15.7 & 22.7 & 14.8 & 941.9 & 94.0 \\
19 & 3000 & 2984.0 & 909.5 & 332 & 6.1 & 11.9 & 15.8 & 25.2 & 14.5 & 916.2 & 91.0 \\
20 & 3494 & 3478.0 & 1060.1 & 317 & 6.2 & 12.0 & 15.4 & 26.3 & 13.8 & 900.0 & 90.0 \\
21 & 3862 & 3846.0 & 1172.3 & 304 & 6.2 & 12.0 & 15.0 & 27.0 & 13.4 & 888.4 & 90.0 \\
22 & 4000 & 3984.0 & 1214.3 & 301 & 5.9 & 11.5 & 14.8 & 27.2 & 13.2 & 884.0 & 90.0 \\
24 & 5000 & 4984.0 & 1519.1 & 278 & 6.5 & 12.6 & 13.0 & 28.2 & 11.4 & 852.8 & 90.0 \\
25 & 5082 & 5066.0 & 1544.1 & 278 & 6.7 & 13.0 & 12.9 & 28.4 & 11.3 & 850.0 & 90.0 \\
26 & 5971 & 5955.0 & 1815.1 & 277 & 7.7 & 15.0 & 11.4 & 29.4 & 9.4 & 823.4 & 88.0
\end{bmatrix}$$

$$d19 := \begin{bmatrix}
13 & 1000 & 984.0 & 299.9 & 254 & 7.5 & 14.6 & 25.6 & 30.0 & 20.1 & 981.0 & 72.0 \\
16 & 1926 & 1910.0 & 582.2 & 246 & 8.2 & 16.0 & 23.6 & 30.5 & 17.8 & 950.0 & 70.0 \\
17 & 2000 & 1984.0 & 604.7 & 245 & 8.2 & 16.0 & 23.5 & 30.6 & 17.5 & 947.7 & 69.0 \\
19 & 3000 & 2984.0 & 909.5 & 232 & 9.3 & 18.0 & 21.6 & 31.4 & 15.1 & 915.2 & 66.0 \\
20 & 3472 & 3456.0 & 1053.4 & 228 & 9.8 & 19.0 & 20.8 & 32.0 & 14.8 & 900.0 & 68.0 \\
21 & 4000 & 3984.0 & 1214.3 & 224 & 10.0 & 19.5 & 19.8 & 32.5 & 13.9 & 883.6 & 69.0 \\
22 & 5000 & 4984.0 & 1519.1 & 218 & 10.3 & 20.0 & 17.7 & 33.4 & 13.3 & 852.9 & 76.0 \\
23 & 5088 & 5072.0 & 1545.9 & 218 & 10.3 & 20.0 & 17.4 & 33.4 & 13.1 & 850.0 & 76.0 \\
24 & 5602 & 5586.0 & 1702.6 & 216 & 10.3 & 20.0 & 16.4 & 33.8 & 11.9 & 834.9 & 75.0 \\
25 & 6000 & 5984.0 & 1823.9 & 215 & 10.4 & 20.2 & 15.8 & 34.3 & 10.8 & 823.1 & 73.0 \\
26 & 6153 & 6137.0 & 1870.6 & 214 & 10.3 & 20.0 & 15.5 & 34.4 & 10.4 & 818.6 & 72.0 \\
27 & 6782 & 6766.0 & 2062.3 & 214 & 10.3 & 20.0 & 14.1 & 34.7 & 7.4 & 800.0 & 64.0
\end{bmatrix}$$

$$d2 := \begin{bmatrix}
4 & 1000 & 984.0 & 299.9 & 224 & 6.2 & 12.0 & 28.7 & 28.9 & 21.6 & 977.7 & 66.0 \\
7 & 2000 & 1984.0 & 604.7 & 232 & 6.4 & 12.4 & 26.6 & 27.0 & 19.6 & 944.8 & 65.0 \\
10 & 3000 & 2984.0 & 909.5 & 238 & 6.1 & 11.8 & 23.7 & 24.3 & 18.3 & 912.8 & 72.0 \\
13 & 4000 & 3984.0 & 1214.3 & 248 & 5.7 & 11.0 & 20.6 & 21.4 & 17.9 & 881.5 & 84.0 \\
16 & 5000 & 4984.0 & 1519.1 & 261 & 5.4 & 10.5 & 18.1 & 19.0 & 15.9 & 851.0 & 87.0 \\
19 & 6000 & 5984.0 & 1823.9 & 275 & 5.2 & 10.1 & 16.6 & 17.6 & 11.5 & 821.3 & 72.0 \\
22 & 7000 & 6984.0 & 2128.7 & 285 & 4.8 & 9.3 & 15.6 & 16.7 & 4.5 & 792.5 & 48.0
\end{bmatrix}$$

$$d21 := \begin{bmatrix} 11 & 1000 & 984.0 & 299.9 & 74 & 3.7 & 7.2 & 20.6 & 23.7 & 15.6 & 987.2 & 73.0 \\ 14 & 2000 & 1984.0 & 604.7 & 82 & 3.0 & 5.8 & 17.4 & 23.4 & 14.5 & 652.9 & 83.0 \\ 15 & 2083 & 2067.0 & 630.0 & 84 & 3.1 & 6.0 & 17.1 & 23.4 & 14.5 & 950.0 & 85.0 \\ 16 & 2261 & 2245.0 & 684.3 & 87 & 2.6 & 5.0 & 16.5 & 23.3 & 14.5 & 944.1 & 88.0 \\ 17 & 2832 & 2816.0 & 858.3 & 106 & 2.1 & 4.0 & 15.0 & 23.5 & 14.3 & 925.0 & 96.0 \\ 18 & 3000 & 2984.0 & 909.5 & 115 & 1.9 & 3.6 & 14.4 & 23.3 & 13.7 & 919.5 & 95.0 \\ 19 & 3219 & 3203.0 & 976.3 & 131 & 1.5 & 3.0 & 13.7 & 23.2 & 12.9 & 912.3 & 95.0 \\ 20 & 3589 & 3573.0 & 1089.1 & 149 & 1.0 & 2.0 & 12.9 & 23.5 & 12.0 & 900.0 & 95.0 \\ 21 & 3786 & 3770.0 & 1149.1 & 166 & 1.0 & 2.0 & 12.5 & 23.6 & 11.6 & 893.8 & 94.0 \\ 24 & 4000 & 3984.0 & 1214.3 & 225 & 1.0 & 1.9 & 12.0 & 23.7 & 11.2 & 886.9 & 95.0 \\ 27 & 4907 & 4891.0 & 1490.8 & 304 & 3.6 & 7.0 & 11.4 & 25.2 & 2.1 & 858.2 & 53.0 \\ 28 & 5161 & 5145.0 & 1568.2 & 303 & 5.1 & 10.0 & 11.7 & 26.1 & -2.4 & 850.0 & 39.0 \\ 29 & 6000 & 5984.0 & 1823.9 & 298 & 8.4 & 16.3 & 11.0 & 27.7 & -7.1 & 824.7 & 27.0 \\ 30 & 6819 & 6803.0 & 2073.6 & 290 & 10.3 & 20.0 & 9.7 & 28.9 & -9.2 & 800.0 & 26.0 \end{bmatrix}$$

$$d23 := \begin{bmatrix} 12 & 1000 & 984.0 & 299.9 & 233 & 4.1 & 8.0 & 25.5 & 30.1 & 21.9 & 982.5 & 81.0 \\ 13 & 1142 & 1126.0 & 343.2 & 231 & 4.1 & 8.0 & 25.7 & 30.7 & 21.6 & 977.8 & 78.0 \\ 15 & 1686 & 1670.0 & 509.0 & 227 & 4.6 & 9.0 & 25.4 & 31.8 & 20.1 & 959.6 & 72.0 \\ 16 & 2000 & 1684.0 & 604.7 & 225 & 5.0 & 9.7 & 24.8 & 32.1 & 19.8 & 949.3 & 74.0 \\ 17 & 2606 & 2590.0 & 789.4 & 225 & 5.1 & 10.0 & 23.7 & 32.7 & 18.9 & 929.6 & 75.0 \\ 18 & 3000 & 2984.0 & 909.5 & 225 & 5.0 & 9.7 & 22.6 & 32.8 & 18.4 & 916.9 & 77.0 \\ 19 & 3528 & 3512.0 & 1070.5 & 223 & 4.6 & 9.0 & 21.4 & 33.1 & 17.9 & 900.0 & 81.0 \\ 20 & 4000 & 3984.0 & 1214.3 & 223 & 4.0 & 7.8 & 20.1 & 33.2 & 17.5 & 885.4 & 85.0 \\ 21 & 4717 & 4701.0 & 1432.9 & 226 & 3.1 & 6.0 & 18.2 & 33.3 & 16.5 & 863.3 & 90.0 \\ 22 & 5000 & 4984.0 & 1519.1 & 230 & 2.5 & 4.8 & 17.9 & 33.7 & 15.3 & 854.7 & 85.0 \\ 23 & 5147 & 5131.0 & 1563.9 & 233 & 2.1 & 4.0 & 17.7 & 33.9 & 14.7 & 850.0 & 83.0 \\ 25 & 6000 & 5984.0 & 1823.9 & 256 & 1.2 & 2.4 & 16.3 & 34.7 & 11.7 & 824.9 & 74.0 \\ 26 & 6323 & 6307.0 & 1922.4 & 269 & 1.0 & 2.0 & 15.5 & 34.8 & 10.8 & 815.4 & 74.0 \\ 29 & 6844 & 6828.0 & 2081.2 & 328 & 1.0 & 1.9 & 14.9 & 35.5 & 7.1 & 800.0 & 59.0 \end{bmatrix}$$

Renaming data elements:

$i16 := 1 .. rows(d16)$

$ML16_{i16} := d16_{i16,1}$

$alt16_{i16} := d16_{i16,4}$

$wd16_{i16} := d16_{i16,5}$

$ws16_{i16} := d16_{i16,6}$

$t16_{i16} := \left(d16_{i16,8} + 273\right)$

$pt16_{i16} := d16_{i16,9}$

$p16_{i16} := d16_{i16,11}$

$rh16_{i16} := d16_{i16,12}$

$i19 := 1 .. rows(d19)$ $\quad$ $i2 := 1 .. rows(d2)$ $\quad$ $i21 := 1 .. rows(d21)$ $\quad$ $i23 := 1 .. rows(d23)$

$ML19_{i19} := d19_{i19,1}$ $\quad$ $ML2_{i2} := d2_{i2,1}$ $\quad$ $ML21_{i21} := d21_{i21,1}$ $\quad$ $ML23_{i23} := d23_{i23,1}$

$alt19_{i19} := d19_{i19,4}$ $\quad$ $alt2_{i2} := d2_{i2,4}$ $\quad$ $alt21_{i21} := d21_{i21,4}$ $\quad$ $alt23_{i23} := d23_{i23,4}$

$wd19_{i19} := d19_{i19,5}$ $\quad$ $wd2_{i2} := d2_{i2,5}$ $\quad$ $wd21_{i21} := d21_{i21,5}$ $\quad$ $wd23_{i23} := d23_{i23,5}$

$ws19_{i19} := d19_{i19,6}$ $\quad$ $ws2_{i2} := d2_{i2,6}$ $\quad$ $ws21_{i21} := d21_{i21,6}$ $\quad$ $ws23_{i23} := d23_{i23,6}$

$t19_{i19} := \left(d19_{i19,8} + 273\right)$ $\quad$ $t2_{i2} := \left(d2_{i2,8} + 273\right)$ $\quad$ $t21_{i21} := \left(d21_{i21,8} + 273\right)$ $\quad$ $t23_{i23} := \left(d23_{i23,8} + 273\right)$

$pt19_{i19} := d19_{i19,9}$ $\quad$ $pt2_{i2} := d2_{i2,9}$ $\quad$ $pt21_{i21} := d21_{i21,9}$ $\quad$ $pt23_{i23} := d23_{i23,9}$

$p19_{i19} := d19_{i19,11}$ $\quad$ $p2_{i2} := d2_{i2,11}$ $\quad$ $p21_{i21} := d21_{i21,11}$ $\quad$ $p23_{i23} := d23_{i23,11}$

$rh19_{i19} := d19_{i19,12}$ $\quad$ $rh2_{i2} := d2_{i2,12}$ $\quad$ $rh21_{i21} := d21_{i21,12}$ $\quad$ $rh23_{i23} := d23_{i23,12}$

Potential Temperature Lapse Rate Calculations:

$j16 := 2 .. rows(d16) - 1$ $\quad$ $j19 := 2 .. rows(d19) - 1$ $\quad$ $j2 := 2 .. rows(d2) - 1$

$$d\Theta dz16_{j16} := \frac{pt16_{j16+1} - pt16_{j16}}{alt16_{j16+1} - alt16_{j16}} \qquad d\Theta dz19_{j19} := \frac{pt19_{j19+1} - pt19_{j19}}{alt19_{j19+1} - alt19_{j19}} \qquad d\Theta dz2_{j2} := \frac{pt2_{j2+1} - pt2_{j2}}{alt2_{j2+1} - alt2_{j2}}$$

$j21 := 2 .. rows(d21) - 1$ $\quad$ $j23 := 2 .. rows(d23) - 1$

$$d\Theta dz21_{j21} := \frac{pt21_{j21+1} - pt21_{j21}}{alt21_{j21+1} - alt21_{j21}} \qquad d\Theta dz23_{j23} := \frac{pt23_{j23+1} - pt23_{j23}}{alt23_{j23+1} - alt23_{j23}}$$

$\gamma 2 := d\Theta dz2$ $\quad$ $\gamma 16 := d\Theta dz16$ $\quad$ $\gamma 19 := d\Theta dz19$ $\quad$ $\gamma 21 := d\Theta dz21$ $\quad$ $\gamma 23 := d\Theta dz23$

$\Theta 2_{j2} := \gamma 2_{j2} \cdot \left(alt2_{j2+1} - alt2_{j2}\right)$ $\quad$ $\Theta 16_{j16} := \gamma 16_{j16} \cdot \left(alt16_{j16+1} - alt16_{j16}\right)$ $\quad$ $\Theta 19_{j19} := \gamma 19_{j19} \cdot \left(alt19_{j19+1} - alt19_{j19}\right)$

$\Theta 21_{j21} := \gamma 21_{j21} \cdot \left(alt21_{j21+1} - alt21_{j21}\right)$ $\quad$ $\Theta 23_{j23} := \gamma 23_{j23} \cdot \left(alt23_{j23+1} - alt23_{j23}\right)$

Calculation of mean stability factors (Θ) for high elevations (300m - 2000m)

$\Theta2^T = (0 \;\; -2.7 \;\; -2.9 \;\; -2.4 \;\; -1.4 \;\; -0.9)$

$\Theta16^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	1	0.4	0.3	2.5	1.1	0.7	0.2	1	0.2	1

$\Theta19^T =$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0.1	0.8	0.6	0.5	0.9	0	0.4	0.5	0.1	0.3

$\Theta21^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0	-0.1	0.2	-0.2	-0.1	0.3	0.1	0.1	1.5	0.9	1.6	1.2

$\Theta23^T =$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	1.1	0.3	0.6	0.1	0.3	0.1	0.1	0.4	0.2	0.8	0.1	0.7

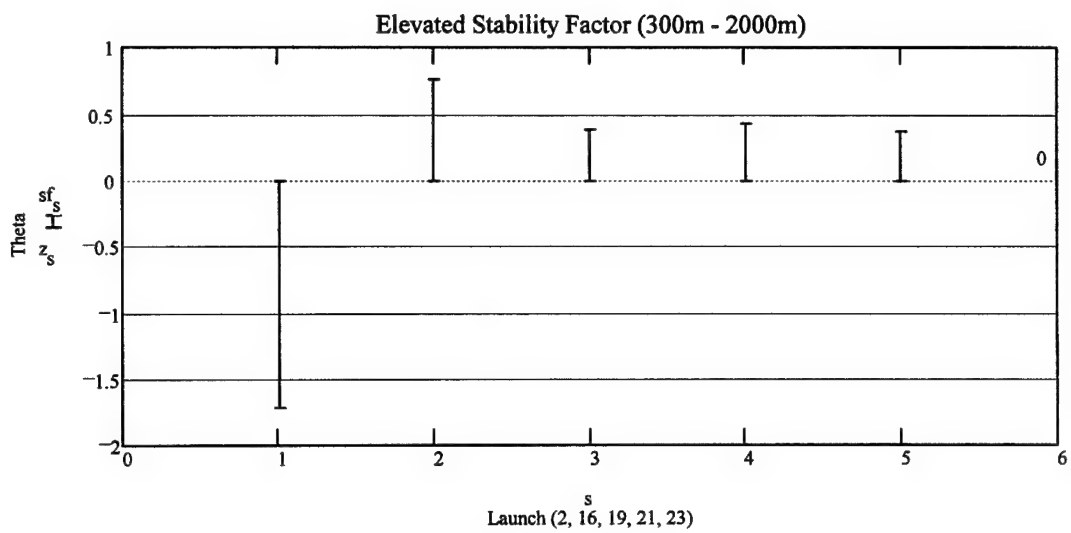
$sf2 := \text{mean}(\Theta2)$ $\quad sf16 := \text{mean}(\Theta16)$ $\quad sf19 := \text{mean}(\Theta19)$ $\quad sf21 := \text{mean}(\Theta21)$ $\quad sf23 := \text{mean}(\Theta23)$

$sf2 = -1.717$ $\quad sf16 = 0.764$ $\quad sf19 = 0.382$ $\quad sf21 = 0.423$ $\quad sf23 = 0.369$

$sft := (sf2 \;\; sf16 \;\; sf19 \;\; sf21 \;\; sf23)$ $\quad sf := sft^T$ $\quad zt := (0 \;\; 0 \;\; 0 \;\; 0 \;\; 0)$

$sft = (-1.717 \;\; 0.764 \;\; 0.382 \;\; 0.423 \;\; 0.369)$ $\quad z := zt^T$

$s := 1 .. 5$



Data for first 600 meters: ORIGIN ≡ 1

$$d16 := \begin{bmatrix} 5 & 199 & 183.0 & 55.8 & 337-360 & 7.2 & 14.0 & 20.0 & 21.0 & 16.3 & 1012.2 & 79.0 \\ 9 & 541 & 525.0 & 160.0 & 5 & 7.2 & 14.0 & 18.9 & 21.0 & 16.5 & 1000.0 & 86.0 \\ 12 & 1000 & 984.0 & 299.9 & 10 & 7.4 & 14.4 & 17.5 & 20.9 & 16.2 & 983.9 & 92.0 \\ 13 & 1114 & 1098.0 & 334.7 & 11 & 7.2 & 14.0 & 17.2 & 21.0 & 16.1 & 979.9 & 93.0 \\ 15 & 1663 & 1647.0 & 502.0 & 10 & 7.2 & 14.0 & 17.0 & 22.0 & 12.5 & 961.0 & 75.0 \\ 16 & 2000 & 1984.0 & 604.7 & 4 & 6.5 & 12.6 & 16.2 & 22.4 & 13.8 & 949.5 & 86.0 \end{bmatrix}$$

$$d19 := \begin{bmatrix} 6 & 219 & 203.0 & 61.9 & 255 & 6.7 & 13.0 & 25.9 & 28.5 & 23.1 & 1007.7 & 85.0 \\ 9 & 442 & 426.0 & 129.8 & 255 & 6.7 & 13.0 & 26.2 & 29.2 & 21.8 & 1000.0 & 77.0 \\ 11 & 624 & 608.0 & 185.3 & 255 & 7.2 & 14.0 & 26.5 & 29.9 & 20.7 & 993.8 & 71.0 \\ 13 & 1000 & 984.0 & 299.9 & 254 & 7.5 & 14.6 & 25.6 & 30.0 & 20.1 & 981.0 & 72.0 \\ 16 & 1926 & 1910.0 & 582.2 & 246 & 8.2 & 16.0 & 23.6 & 30.5 & 17.8 & 950.0 & 70.0 \\ 17 & 2000 & 1984.0 & 604.7 & 245 & 8.2 & 16.0 & 23.5 & 30.6 & 17.5 & 947.7 & 69.0 \end{bmatrix}$$

$$d2 := \begin{pmatrix} 1 & 344 & 328 & 100.0 & 180 & 6.1 & 11.7 & 28.5 & 28.7 & 23.4 & 1000 & 78 \\ 4 & 1000 & 984.0 & 299.9 & 224 & 6.2 & 12.0 & 28.7 & 28.9 & 21.6 & 977.7 & 66.0 \\ 7 & 2000 & 1984.0 & 604.7 & 232 & 6.4 & 12.4 & 26.6 & 27.0 & 19.6 & 944.8 & 65.0 \end{pmatrix}$$

$$d21 := \begin{bmatrix} 5 & 212 & 196.0 & 59.7 & 69 & 3.6 & 7.0 & 22.8 & 24.0 & 18.7 & 1014.8 & 78.0 \\ 9 & 631 & 615.0 & 187.5 & 73 & 3.6 & 7.0 & 21.6 & 23.7 & 16.3 & 1000.0 & 72.0 \\ 11 & 1000 & 984.0 & 299.9 & 74 & 3.7 & 7.2 & 20.6 & 23.7 & 15.6 & 987.2 & 73.0 \\ 14 & 2000 & 1984.0 & 604.7 & 82 & 3.0 & 5.8 & 17.4 & 23.4 & 14.5 & 652.9 & 83.0 \end{bmatrix}$$

$$d23 := \begin{bmatrix} 6 & 250 & 234.0 & 71.3 & 257 & 3.1 & 6.0 & 25.5 & 27.9 & 22.7 & 1008.3 & 84.0 \\ 9 & 488 & 472.0 & 143.9 & 246 & 3.6 & 7.0 & 25.3 & 28.4 & 22.5 & 1000.0 & 85.0 \\ 12 & 1000 & 984.0 & 299.9 & 233 & 4.1 & 8.0 & 25.5 & 30.1 & 21.9 & 982.5 & 81.0 \\ 13 & 1142 & 1126.0 & 343.2 & 231 & 4.1 & 8.0 & 25.7 & 30.7 & 21.6 & 977.8 & 78.0 \\ 15 & 1686 & 1670.0 & 509.0 & 227 & 4.6 & 9.0 & 25.4 & 31.8 & 20.1 & 959.6 & 72.0 \\ 16 & 2000 & 1684.0 & 604.7 & 225 & 5.0 & 9.7 & 24.8 & 32.1 & 19.8 & 949.3 & 74.0 \end{bmatrix}$$

i16 := 1 .. rows(d16)   i19 := 1 .. rows(d19)   i2 := 1 .. rows(d2)   i21 := 1 .. rows(d21)   i23 := 1 .. rows(d23)

i16 := 1 .. rows(d16)

Renaming data elements:

$ML16_{i16} := d16_{i16,1}$ $\quad dt16_{i16} := d16_{i16,10}$ $\quad rh16_{i16} := d16_{i16,12}$

$alt16_{i16} := d16_{i16,4}$ $\quad t16_{i16} := d16_{i16,8}$

$wd16_{i16} := d16_{i16,5}$ $\quad pt16_{i16} := d16_{i16,9}$

$ws16_{i16} := d16_{i16,6}$ $\quad p16_{i16} := d16_{i16,11}$

$i19 := 1 .. rows(d19)$ $\quad i2 := 1 .. rows(d2)$ $\quad i21 := 1 .. rows(d21)$ $\quad i23 := 1 .. rows(d23)$

$ML19_{i19} := d19_{i19,1}$ $\quad ML2_{i2} := d2_{i2,1}$ $\quad ML21_{i21} := d21_{i21,1}$ $\quad ML23_{i23} := d23_{i23,1}$

$alt19_{i19} := d19_{i19,4}$ $\quad alt2_{i2} := d2_{i2,4}$ $\quad alt21_{i21} := d21_{i21,4}$ $\quad alt23_{i23} := d23_{i23,4}$

$wd19_{i19} := d19_{i19,5}$ $\quad wd2_{i2} := d2_{i2,5}$ $\quad wd21_{i21} := d21_{i21,5}$ $\quad wd23_{i23} := d23_{i23,5}$

$ws19_{i19} := d19_{i19,6}$ $\quad ws2_{i2} := d2_{i2,6}$ $\quad ws21_{i21} := d21_{i21,6}$ $\quad ws23_{i23} := d23_{i23,6}$

$t19_{i19} := d19_{i19,8}$ $\quad t2_{i2} := d2_{i2,8}$ $\quad t21_{i21} := d21_{i21,8}$ $\quad t23_{i23} := d23_{i23,8}$

$pt19_{i19} := d19_{i19,9}$ $\quad pt2_{i2} := d2_{i2,9}$ $\quad pt21_{i21} := d21_{i21,9}$ $\quad pt23_{i23} := d23_{i23,9}$

$p19_{i19} := d19_{i19,11}$ $\quad p2_{i2} := d2_{i2,11}$ $\quad p21_{i21} := d21_{i21,11}$ $\quad p23_{i23} := d23_{i23,11}$

$rh19_{i19} := d19_{i19,12}$ $\quad rh2_{i2} := d2_{i2,12}$ $\quad rh21_{i21} := d21_{i21,12}$ $\quad rh23_{i23} := d23_{i23,12}$

$dt19_{i19} := d19_{i19,10}$ $\quad dt2_{i2} := d2_{i2,10}$ $\quad dt21_{i21} := d21_{i21,10}$ $\quad dt23_{i23} := d23_{i23,10}$

Potential Temperature Lapse Rate Calculations:

$j16 := 1 .. rows(d16) - 1$ $\quad j19 := 1 .. rows(d19) - 1$ $\quad j2 := 1 .. rows(d2) - 1$

$$d\Theta dz16_{j16} := \frac{pt16_{j16+1} - pt16_{j16}}{alt16_{j16+1} - alt16_{j16}} \qquad d\Theta dz19_{j19} := \frac{pt19_{j19+1} - pt19_{j19}}{alt19_{j19+1} - alt19_{j19}} \qquad d\Theta dz2_{j2} := \frac{pt2_{j2+1} - pt2_{j2}}{alt2_{j2+1} - alt2_{j2}}$$

$j21 := 1 .. rows(d21) - 1$ $\quad j23 := 1 .. rows(d23) - 1$

$$d\Theta dz21_{j21} := \frac{pt21_{j21+1} - pt21_{j21}}{alt21_{j21+1} - alt21_{j21}} \qquad d\Theta dz23_{j23} := \frac{pt23_{j23+1} - pt23_{j23}}{alt23_{j23+1} - alt23_{j23}}$$

$\gamma 2 := d\Theta dz2$ $\quad \gamma 16 := d\Theta dz16$ $\quad \gamma 19 := d\Theta dz19$ $\quad \gamma 21 := d\Theta dz21$ $\quad \gamma 23 := d\Theta dz23$

Potential Temperature Lapse Rate Mean and Variance Calculations:

$m\_\gamma 2 := mean(\gamma 2)$ $\quad m\_\gamma 2 = -2.617 \cdot 10^{-3}$ $\quad v\_\gamma 2 := var(\gamma 2)$ $\quad v\_\gamma 2 = 1.308 \cdot 10^{-5}$

$m\_\gamma 16 := mean(\gamma 16)$ $\quad m\_\gamma 16 = 2.406 \cdot 10^{-3}$ $\quad v\_\gamma 16 := var(\gamma 16)$ $\quad v\_\gamma 16 = 6.144 \cdot 10^{-6}$

$m\_\gamma19 := mean(\gamma19)$ $m\_\gamma19 = 6.002\cdot10^{-3}$ $v\_\gamma19 := var(\gamma19)$ $v\_\gamma19 = 2.178\cdot10^{-5}$

$m\_\gamma21 := mean(\gamma21)$ $m\_\gamma21 = -1.111\cdot10^{-3}$ $v\_\gamma21 := var(\gamma21)$ $v\_\gamma21 = 9.264\cdot10^{-7}$

$m\_\gamma23 := mean(\gamma23)$ $m\_\gamma23 = 8.282\cdot10^{-3}$ $v\_\gamma23 := var(\gamma23)$ $v\_\gamma23 = 1.381\cdot10^{-5}$

$v\_t\gamma := (v\_\gamma2 \;\; v\_\gamma16 \;\; v\_\gamma19 \;\; v\_\gamma21 \;\; v\_\gamma23)$

$m\_t\gamma := (m\_\gamma2 \;\; m\_\gamma16 \;\; m\_\gamma19 \;\; m\_\gamma21 \;\; m\_\gamma23)$ $m\_\gamma := m\_t\gamma^T$ $v\_\gamma := v\_t\gamma^T$

$v\_t\gamma = (1.308\cdot10^{-5} \;\; 6.144\cdot10^{-6} \;\; 2.178\cdot10^{-5} \;\; 9.264\cdot10^{-7} \;\; 1.381\cdot10^{-5})$

$m\_t\gamma = (-2.617\cdot10^{-3} \;\; 2.406\cdot10^{-3} \;\; 6.002\cdot10^{-3} \;\; -1.111\cdot10^{-3} \;\; 8.282\cdot10^{-3})$

Potential Temperature Mean and Variance Calculations:

$m\_pt2 := mean(pt2)$ $m\_pt2 = 28.2$ $v\_pt2 := var(pt2)$ $v\_pt2 = 0.727$

$m\_pt16 := mean(pt16)$ $m\_pt16 = 21.383$ $v\_pt16 := var(pt16)$ $v\_pt16 = 0.348$

$m\_pt19 := mean(pt19)$ $m\_pt19 = 29.783$ $v\_pt19 := var(pt19)$ $v\_pt19 = 0.538$

$m\_pt21 := mean(pt21)$ $m\_pt21 = 23.7$ $v\_pt21 := var(pt21)$ $v\_pt21 = 0.045$

$m\_pt23 := mean(pt23)$ $m\_pt23 = 30.167$ $v\_pt23 := var(pt23)$ $v\_pt23 = 2.492$

$v\_tpt := (v\_pt2 \;\; v\_pt16 \;\; v\_pt19 \;\; v\_pt21 \;\; v\_pt23)$

$m\_tpt := (m\_pt2 \;\; m\_pt16 \;\; m\_pt19 \;\; m\_pt21 \;\; m\_pt23)$ $m\_pt := m\_tpt^T$ $v\_pt := v\_tpt^T$

$v\_tpt = (0.727 \;\; 0.348 \;\; 0.538 \;\; 0.045 \;\; 2.492)$

$m\_tpt = (28.2 \;\; 21.383 \;\; 29.783 \;\; 23.7 \;\; 30.167)$

Wind Speed Mean and Variance Calculations:

$m\_ws2 := mean(ws2)$ $m\_ws2 = 6.233$ $v\_ws2 := var(ws2)$ $v\_ws2 = 0.016$

$m\_ws16 := mean(ws16)$ $m\_ws16 = 7.117$ $v\_ws16 := var(ws16)$ $v\_ws16 = 0.081$

$m\_ws19 := mean(ws19)$ $m\_ws19 = 7.417$ $v\_ws19 := var(ws19)$ $v\_ws19 = 0.385$

$m\_ws21 := mean(ws21)$ $m\_ws21 = 3.475$ $v\_ws21 := var(ws21)$ $v\_ws21 = 0.077$

$m\_ws23 := mean(ws23)$ $m\_ws23 = 4.083$ $v\_ws23 := var(ws23)$ $v\_ws23 = 0.385$

$v\_tws := (v\_ws2 \;\; v\_ws16 \;\; v\_ws19 \;\; v\_ws21 \;\; v\_ws23)$ $v\_ws := v\_tws^T$

$m\_tws := (m\_ws2 \;\; m\_ws16 \;\; m\_ws19 \;\; m\_ws21 \;\; m\_ws23)$ $m\_ws := m\_tws^T$

$v\_tws = (0.016 \;\; 0.081 \;\; 0.385 \;\; 0.077 \;\; 0.385)$ $m\_tws = (6.233 \;\; 7.117 \;\; 7.417 \;\; 3.475 \;\; 4.083)$

Wind Direction Mean and Variance Calculations:

m_wd2 := mean(wd2)	m_wd2 = 212	v_wd2 := var(wd2)	v_wd2 = 522.667
m_wd16 := mean(wd16)	m_wd16 = 2.833	v_wd16 := var(wd16)	v_wd16 = 140.472
m_wd19 := mean(wd19)	m_wd19 = 251.667	v_wd19 := var(wd19)	v_wd19 = 19.222
m_wd21 := mean(wd21)	m_wd21 = 74.5	v_wd21 := var(wd21)	v_wd21 = 22.25
m_wd23 := mean(wd23)	m_wd23 = 236.5	v_wd23 := var(wd23)	v_wd23 = 129.25

v_twd := (v_wd2 v_wd16 v_wd19 v_wd21 v_wd23) $v\_wd := v\_twd^T$

m_twd := (m_wd2 m_wd16 m_wd19 m_wd21 m_wd23) $m\_wd := m\_twd^T$

v_twd = (523 140 19 22 129)   m_twd = (212 2.8 251.7 74.5 236.5)

Relative Humidity Mean and Variance Calculations:

m_rh2 := mean(rh2)	m_rh2 = 69.667	v_rh2 := var(rh2)	v_rh2 = 34.889
m_rh16 := mean(rh16)	m_rh16 = 85.167	v_rh16 := var(rh16)	v_rh16 = 41.806
m_rh19 := mean(rh19)	m_rh19 = 74	v_rh19 := var(rh19)	v_rh19 = 30.667
m_rh21 := mean(rh21)	m_rh21 = 76.5	v_rh21 := var(rh21)	v_rh21 = 19.25
m_rh23 := mean(rh23)	m_rh23 = 79	v_rh23 := var(rh23)	v_rh23 = 23.333

v_trh := (v_rh2 v_rh16 v_rh19 v_rh21 v_rh23)

m_trh := (m_rh2 m_rh16 m_rh19 m_rh21 m_rh23) $m\_rh := m\_trh^T$ $v\_rh := v\_trh^T$

v_trh = (35 42 31 19 23)   m_trh = (69.7 85.2 74 76.5 79)   s := 1 .. 5

Temperature Mean and Variance Calculations:

m_t2 := mean(t2)	m_t2 = 27.933	v_t2 := var(t2)	v_t2 = 0.896
m_t16 := mean(t16)	m_t16 = 17.8	v_t16 := var(t16)	v_t16 = 1.617
m_t19 := mean(t19)	m_t19 = 25.217	v_t19 := var(t19)	v_t19 = 1.465
m_t21 := mean(t21)	m_t21 = 20.6	v_t21 := var(t21)	v_t21 = 4.02
m_t23 := mean(t23)	m_t23 = 25.367	v_t23 := var(t23)	v_t23 = 0.079

v_tt := (v_t2 v_t16 v_t19 v_t21 v_t23)

m_tt := (m_t2 m_t16 m_t19 m_t21 m_t23) $m\_t := m\_tt^T$ $v\_t := v\_tt^T$

v_tt = (1 2 1 4 0)   m_tt = (27.9 17.8 25.2 20.6 25.4)

Dew Point Temperature Mean and Variance Calculations:

$m\_dt2 := \mathrm{mean}(dt2)$ $m\_dt2 = 21.533$ $v\_dt2 := \mathrm{var}(dt2)$ $v\_dt2 = 2.409$

$m\_dt16 := \mathrm{mean}(dt16)$ $m\_dt16 = 15.233$ $v\_dt16 := \mathrm{var}(dt16)$ $v\_dt16 = 2.326$

$m\_dt19 := \mathrm{mean}(dt19)$ $m\_dt19 = 20.167$ $v\_dt19 := \mathrm{var}(dt19)$ $v\_dt19 = 4.046$

$m\_dt21 := \mathrm{mean}(dt21)$ $m\_dt21 = 16.275$ $v\_dt21 := \mathrm{var}(dt21)$ $v\_dt21 = 2.372$

$m\_dt23 := \mathrm{mean}(dt23)$ $m\_dt23 = 21.433$ $v\_dt23 := \mathrm{var}(dt23)$ $v\_dt23 = 1.239$

$v\_tdt := (v\_dt2 \;\; v\_dt16 \;\; v\_dt19 \;\; v\_dt21 \;\; v\_dt23)$

$m\_tdt := (m\_dt2 \;\; m\_dt16 \;\; m\_dt19 \;\; m\_dt21 \;\; m\_dt23)$ $m\_dt := m\_tdt^T$ $v\_dt := v\_tdt^T$

$v\_tdt = (2 \;\; 2 \;\; 4 \;\; 2 \;\; 1)$ $m\_tdt = (21.5 \;\; 15.2 \;\; 20.2 \;\; 16.3 \;\; 21.4)$

$zt := (0 \;\; 0 \;\; 0 \;\; 0 \;\; 0)$ $z := zt^T$

$$altlr2_{j2} := alt2_{j2} + \left(\frac{alt2_{j2+1} - alt2_{j2}}{2}\right)$$

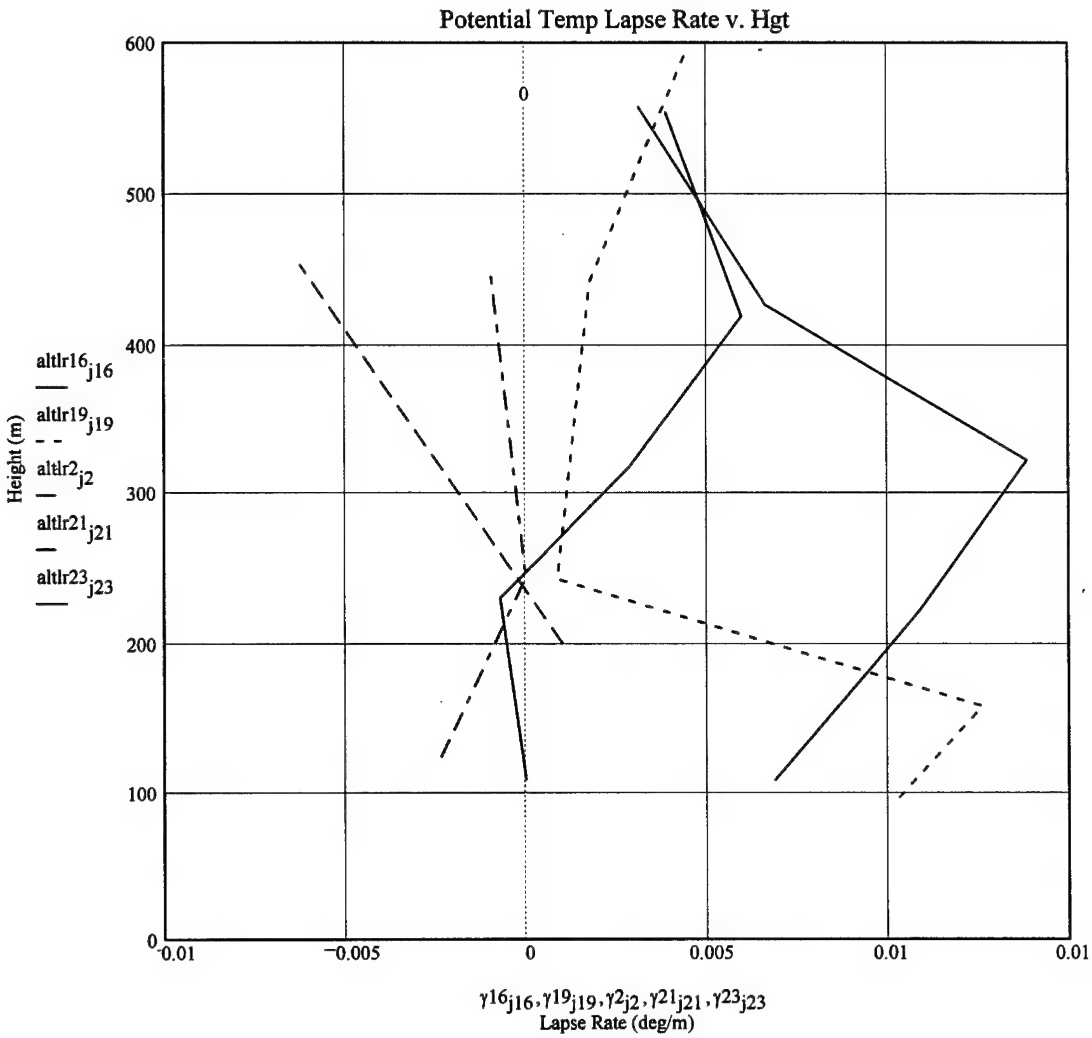
$$altlr16_{j16} := alt16_{j16} + \left(\frac{alt16_{j16+1} - alt16_{j16}}{2}\right)$$

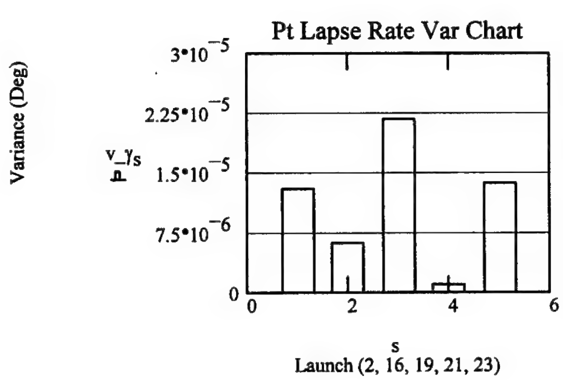
$$altlr19_{j19} := alt19_{j19} + \left(\frac{alt19_{j19+1} - alt19_{j19}}{2}\right)$$

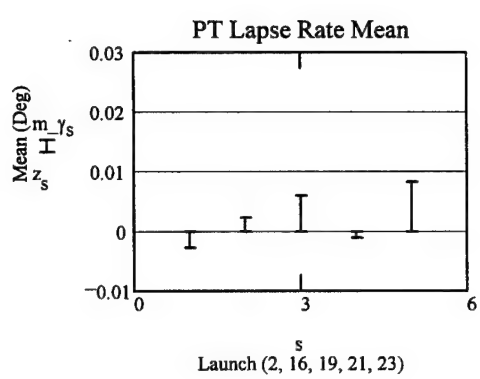
$$altlr21_{j21} := alt21_{j21} + \left(\frac{alt21_{j21+1} - alt21_{j21}}{2}\right)$$

$$altlr23_{j23} := alt23_{j23} + \left(\frac{alt23_{j23+1} - alt23_{j23}}{2}\right)$$

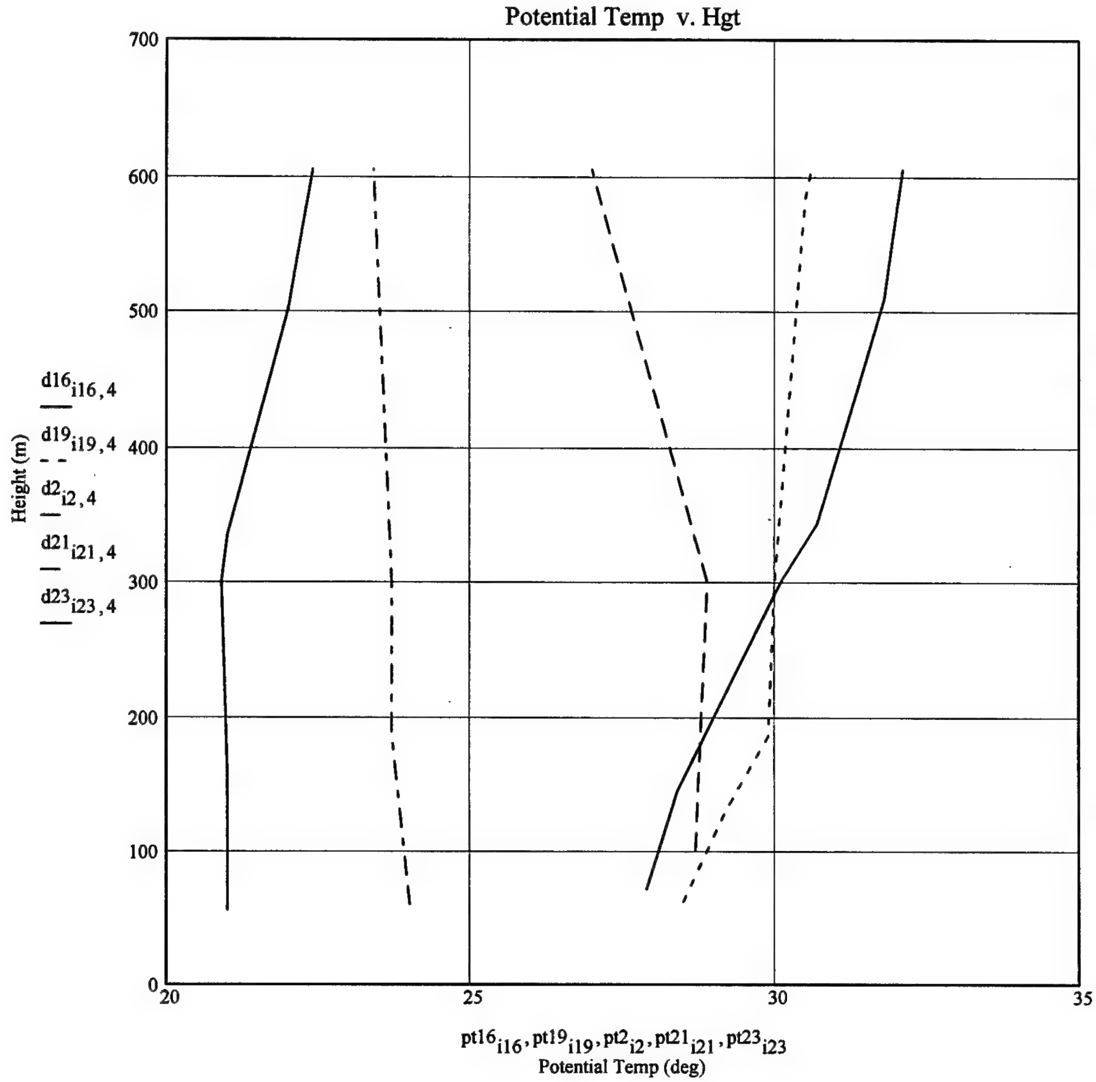
Potential Temperature Lapse Rate Profiles:

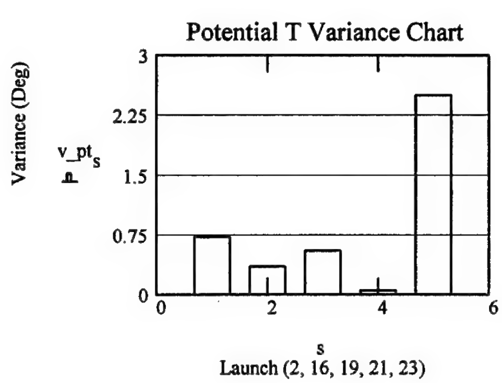


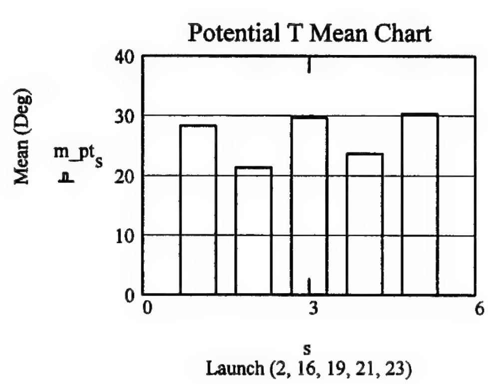




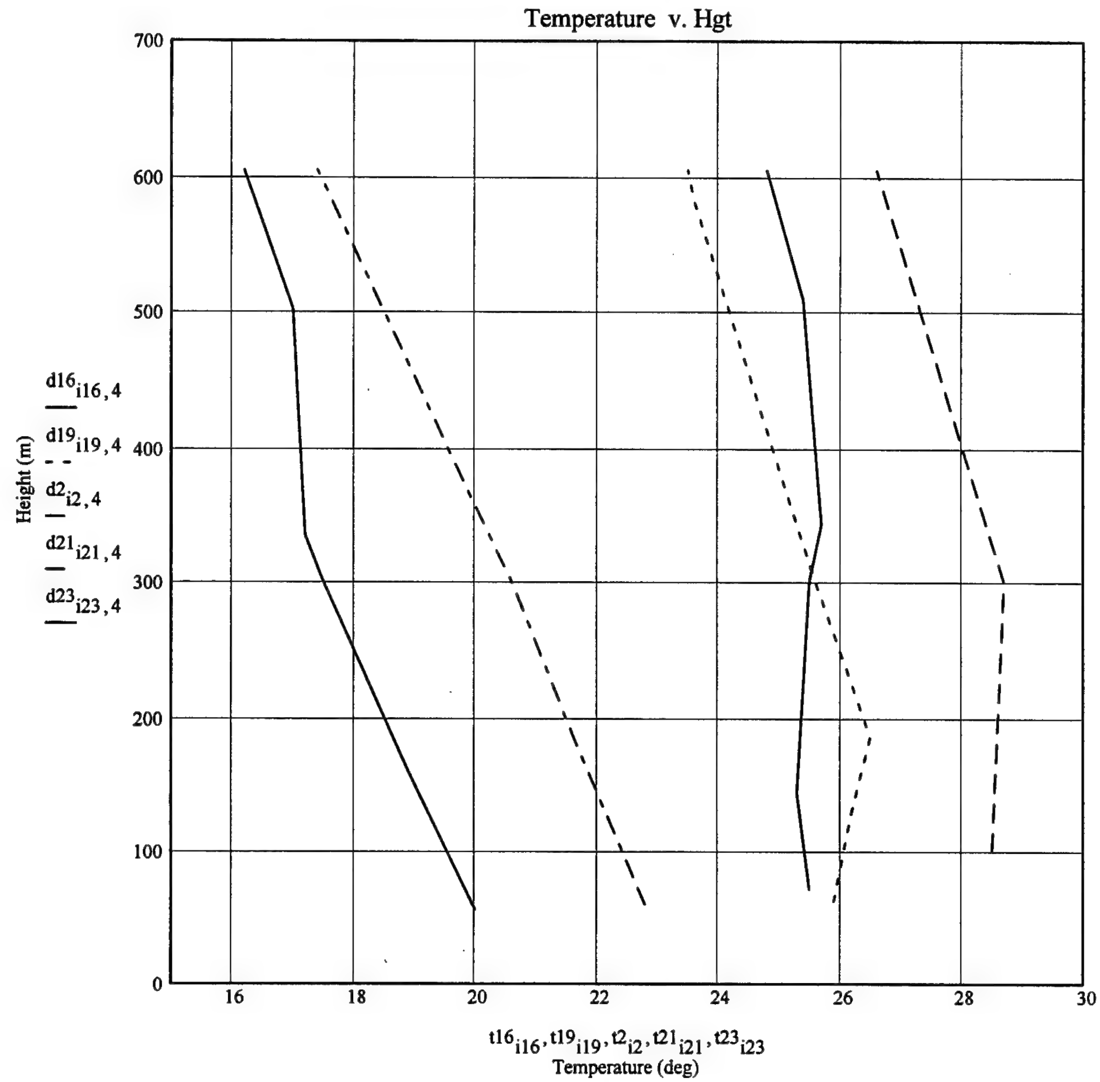
Potential Temperature Profiles:

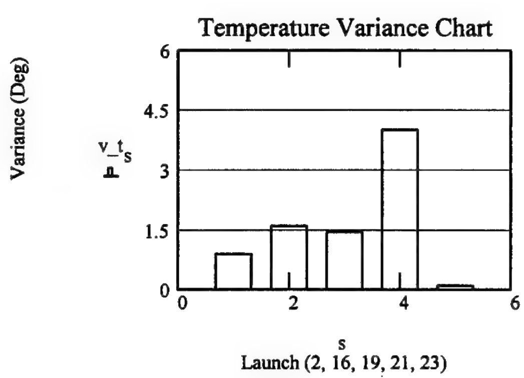


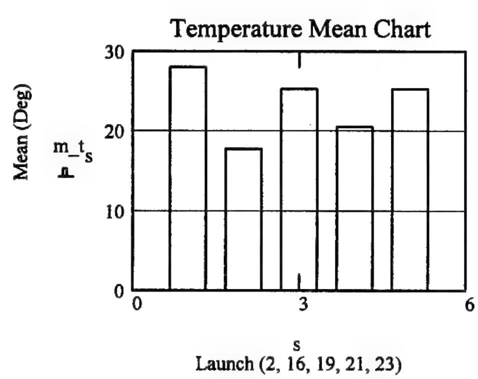




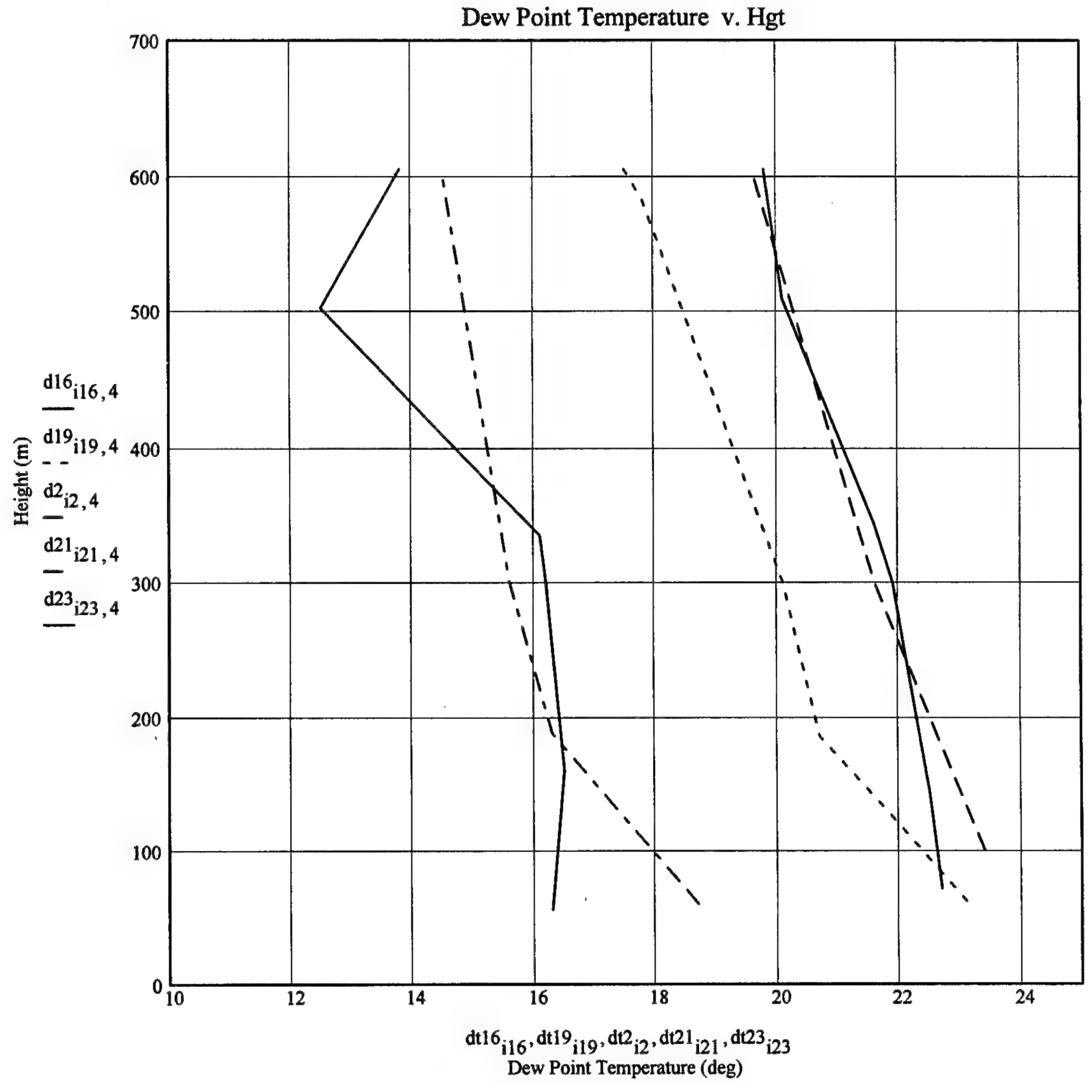
Temperature Profiles:

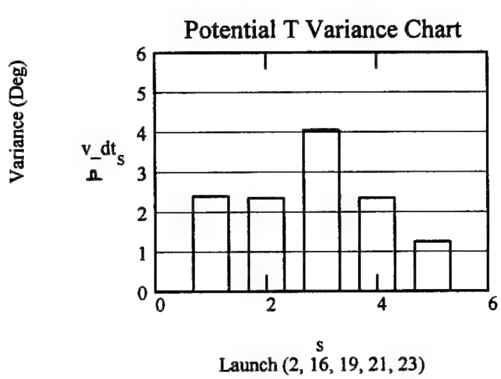


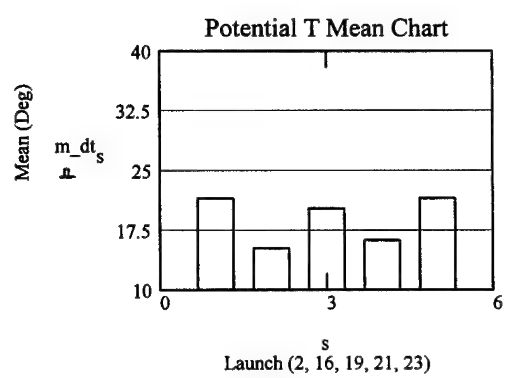




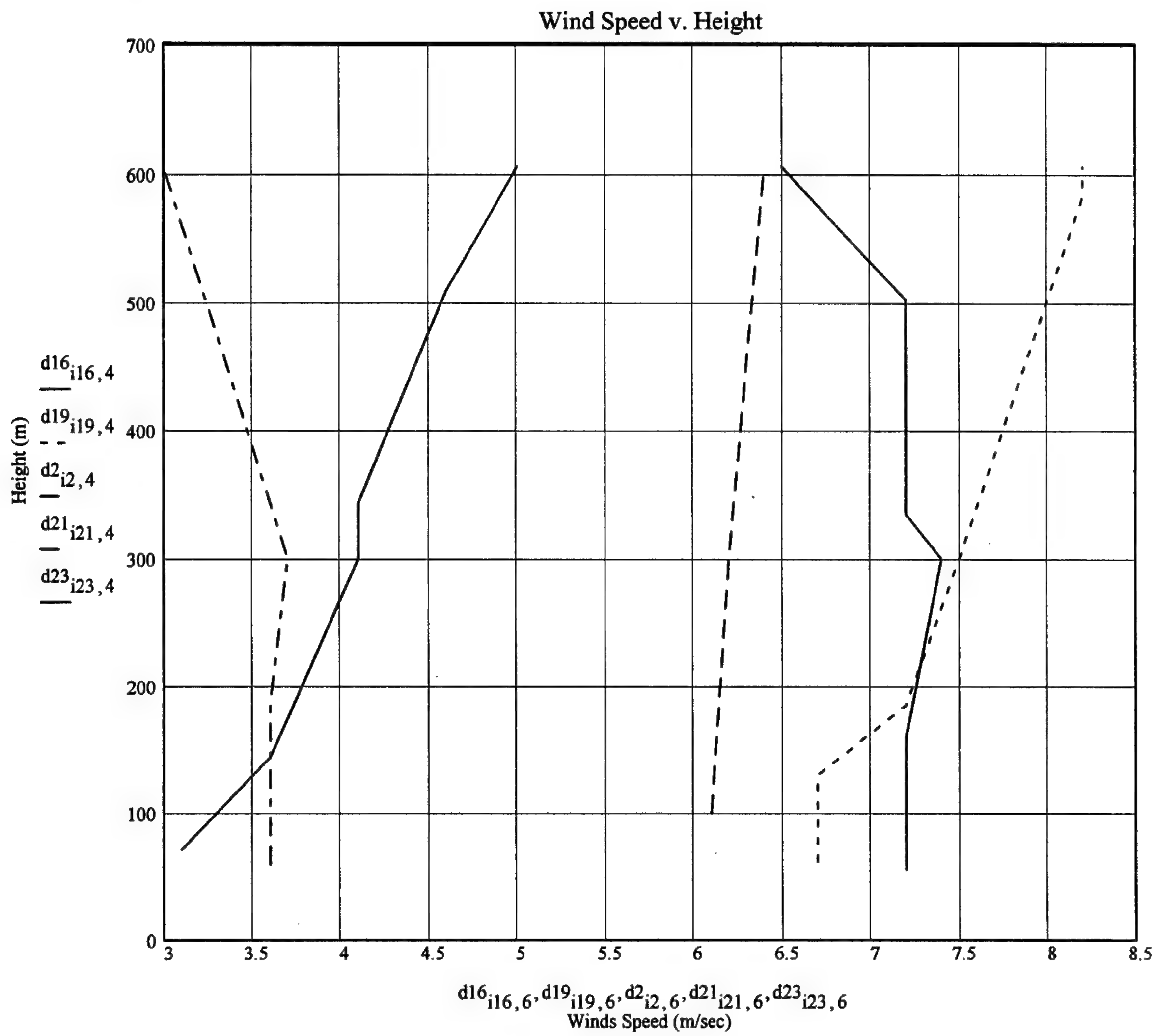
Dewpoint Point Temperature Profiles:

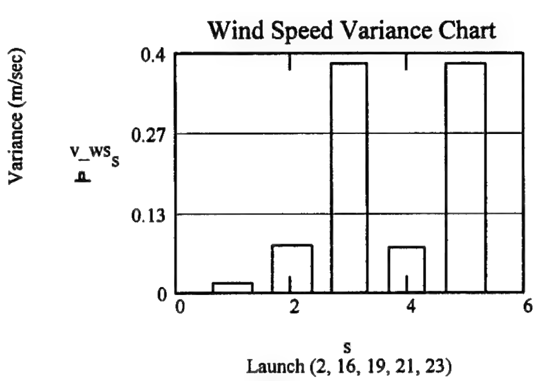


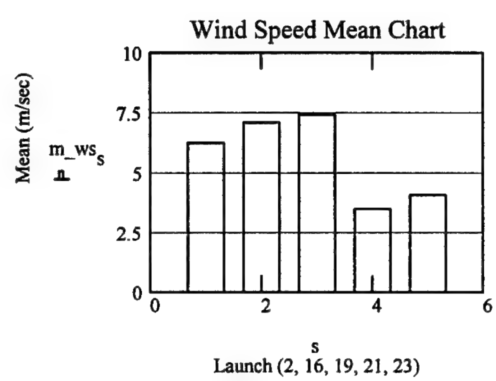




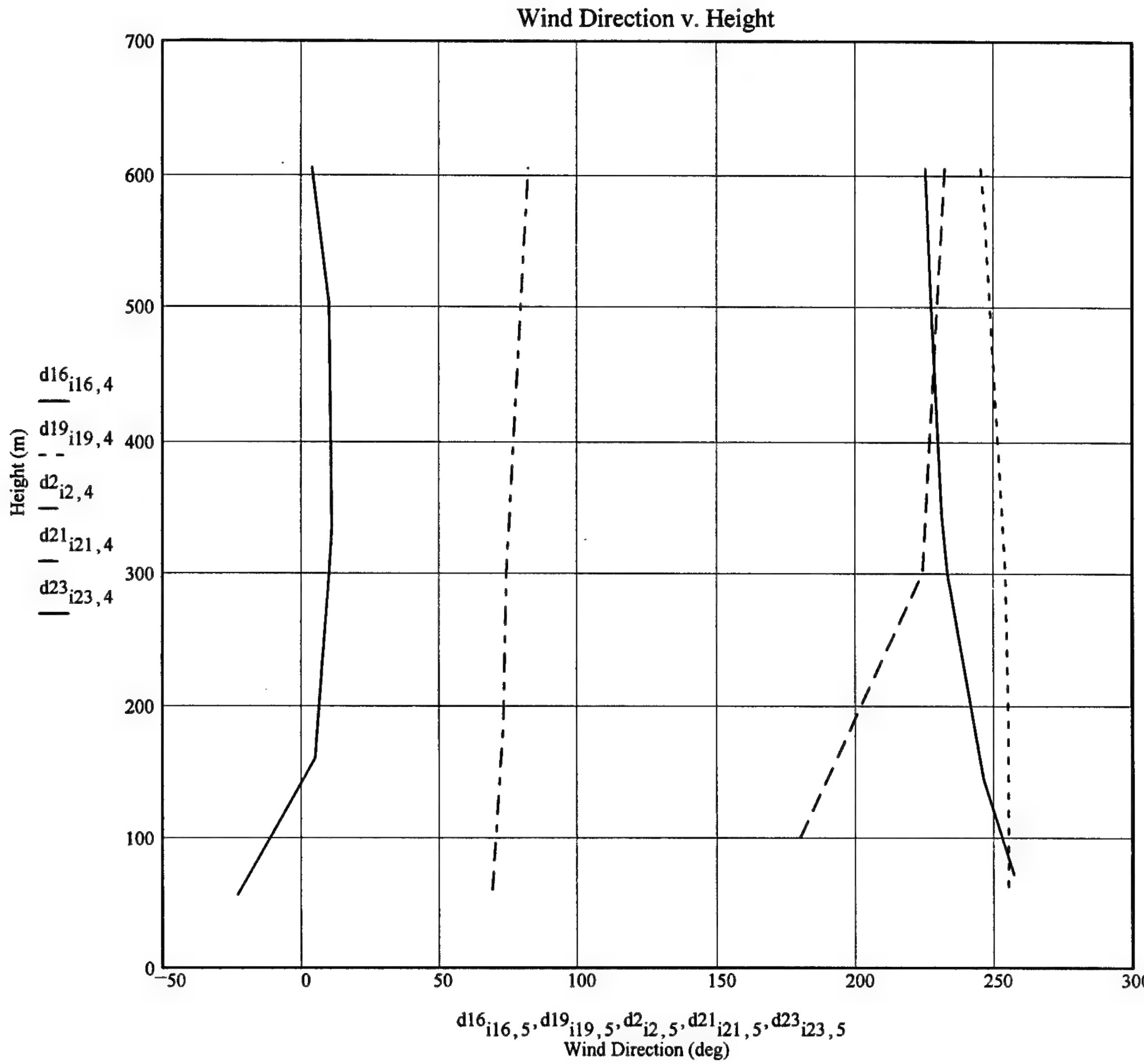
Wind Speed Profiles:

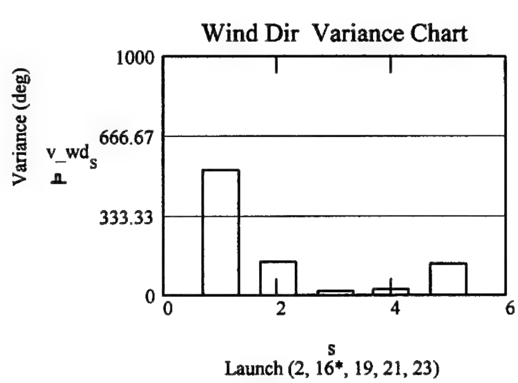


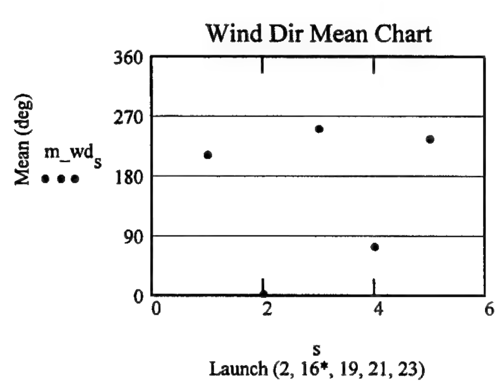




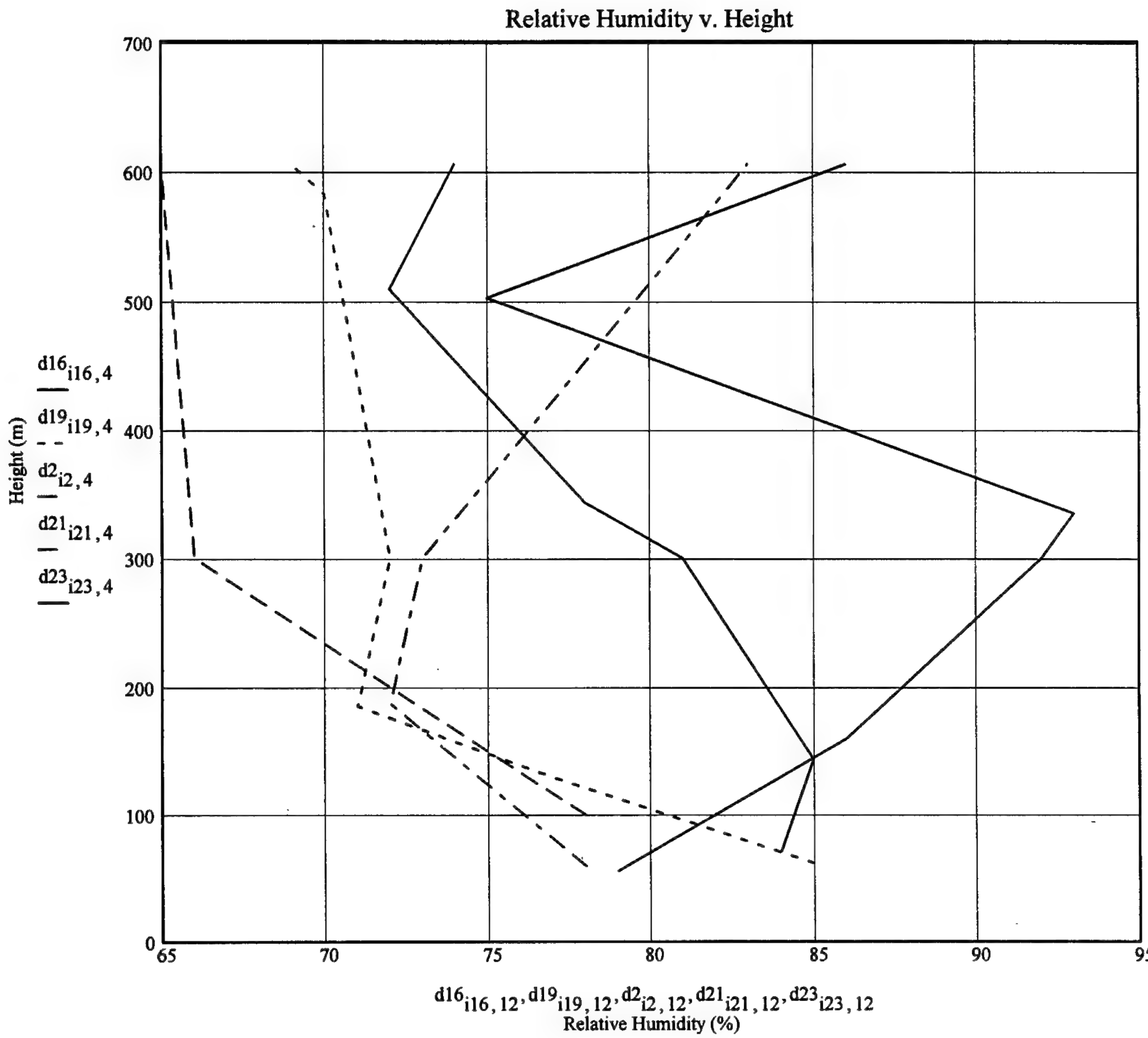
Wind Direction Profiles:

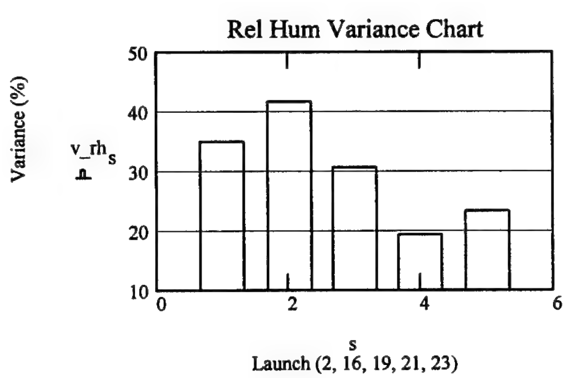


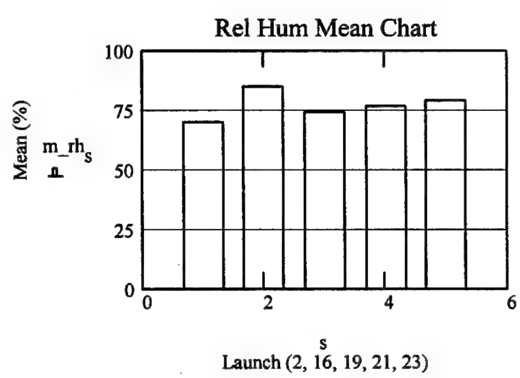




Relative Humidity Profiles:







Data for first 300 meters: ORIGIN ≡ 1

$$d2 := \begin{pmatrix} 1 & 16 & 0.0 & 0.0 & 180 & 2.6 & 5.0 & 28.4 & 28.3 & 25.4 & 1011.0 & 84.0 \\ 4 & 1000 & 984.0 & 299.9 & 224 & 6.2 & 12.0 & 28.7 & 28.9 & 21.6 & 977.7 & 66.0 \end{pmatrix}$$

$$d16 := \begin{bmatrix} 1 & 16 & 0.0 & 0.0 & 20 & 6.2 & 12.0 & 20.8 & 21.3 & 16.3 & 1018.7 & 76.0 \\ 5 & 199 & 183.0 & 55.8 & 337-180 & 7.2 & 14.0 & 20.0 & 21.0 & 16.3 & 1012.2 & 79.0 \\ 9 & 541 & 525.0 & 160.0 & 5 & 7.2 & 14.0 & 18.9 & 21.0 & 16.5 & 1000.0 & 86.0 \\ 12 & 1000 & 984.0 & 299.9 & 10 & 7.4 & 14.4 & 17.5 & 20.9 & 16.2 & 983.9 & 92.0 \end{bmatrix}$$

$i16 := 1 .. \mathrm{rows}(d16)$

$$d19 := \begin{bmatrix} 1 & 16 & 0.0 & 0.0 & 170 & 1.5 & 3.0 & 25.1 & 26.9 & 22.8 & 1014.8 & 87.0 \\ 6 & 219 & 203.0 & 61.9 & 255 & 6.7 & 13.0 & 25.9 & 28.5 & 23.1 & 1007.7 & 85.0 \\ 9 & 442 & 426.0 & 129.8 & 255 & 6.7 & 13.0 & 26.2 & 29.2 & 21.8 & 1000.0 & 77.0 \\ 11 & 624 & 608.0 & 185.3 & 255 & 7.2 & 14.0 & 26.5 & 29.9 & 20.7 & 993.8 & 71.0 \\ 13 & 1000 & 984.0 & 299.9 & 254 & 7.5 & 14.6 & 25.6 & 30.0 & 20.1 & 981.0 & 72.0 \end{bmatrix}$$

$$d21 := \begin{bmatrix} 1 & 16 & 0.0 & 0.0 & 20 & 1.5 & 3.0 & 17.3 & 17.3 & 15.0 & 1021.8 & 86.0 \\ 5 & 212 & 196.0 & 59.7 & 69 & 3.6 & 7.0 & 22.8 & 24.0 & 18.7 & 1014.8 & 78.0 \\ 9 & 631 & 615.0 & 187.5 & 73 & 3.6 & 7.0 & 21.6 & 23.7 & 16.3 & 1000.0 & 72.0 \\ 11 & 1000 & 984.0 & 299.9 & 74 & 3.7 & 7.2 & 20.6 & 23.7 & 15.6 & 987.2 & 73.0 \end{bmatrix}$$

$$d23 := \begin{bmatrix} 1 & 16 & 0.0 & 0.0 & 250 & 2.6 & 5.0 & 27.1 & 29.0 & 23.6 & 1016.4 & 81.0 \\ 6 & 250 & 234.0 & 71.3 & 257 & 3.1 & 6.0 & 25.5 & 27.9 & 22.7 & 1008.3 & 84.0 \\ 9 & 488 & 472.0 & 143.9 & 246 & 3.6 & 7.0 & 25.3 & 28.4 & 22.5 & 1000.0 & 85.0 \\ 12 & 1000 & 984.0 & 299.9 & 233 & 4.1 & 8.0 & 25.5 & 30.1 & 21.9 & 982.5 & 81.0 \end{bmatrix}$$

$ML16_{i16} := d16_{i16,1}$

$alt16_{i16} := d16_{i16,4}$

$wd16_{i16} := d16_{i16,5}$

$ws16_{i16} := d16_{i16,6}$

$t16_{i16} := d16_{i16,8}$

$pt16_{i16} := d16_{i16,9}$

$dt16_{i16} := d16_{i16,10}$

$p16_{i16} := d16_{i16,11}$

$rh16_{i16} := d16_{i16,12}$

Renaming data elements:

$i19 := 1 .. \mathrm{rows}(d19)$

$ML19_{i19} := d19_{i19,1}$

$alt19_{i19} := d19_{i19,4}$

$wd19_{i19} := d19_{i19,5}$

$ws19_{i19} := d19_{i19,6}$

$t19_{i19} := d19_{i19,8}$

$pt19_{i19} := d19_{i19,9}$

$dt19_{i19} := d19_{i19,10}$

$p19_{i19} := d19_{i19,11}$

$rh19_{i19} := d19_{i19,12}$

$i2 := 1 .. \mathrm{rows}(d2)$

$ML2_{i2} := d2_{i2,1}$

$alt2_{i2} := d2_{i2,4}$

$wd2_{i2} := d2_{i2,5}$

$ws2_{i2} := d2_{i2,6}$

$t2_{i2} := d2_{i2,8}$

$pt2_{i2} := d2_{i2,9}$

$dt2_{i2} := d2_{i2,10}$

$p2_{i2} := d2_{i2,11}$

$rh2_{i2} := d2_{i2,12}$

$i21 := 1 .. \mathrm{rows}(d21)$

$ML21_{i21} := d21_{i21,1}$

$alt21_{i21} := d21_{i21,4}$

$wd21_{i21} := d21_{i21,5}$

$ws21_{i21} := d21_{i21,6}$

$t21_{i21} := d21_{i21,8}$

$pt21_{i21} := d21_{i21,9}$

$dt21_{i21} := d21_{i21,10}$

$p21_{i21} := d21_{i21,11}$

$rh21_{i21} := d21_{i21,12}$

$i23 := 1 .. \mathrm{rows}(d23)$

$ML23_{i23} := d23_{i23,1}$

$alt23_{i23} := d23_{i23,4}$

$wd23_{i23} := d23_{i23,5}$

$ws23_{i23} := d23_{i23,6}$

$t23_{i23} := d23_{i23,8}$

$pt23_{i23} := d23_{i23,9}$

$dt23_{i23} := d23_{i23,10}$

$p23_{i23} := d23_{i23,11}$

$rh23_{i23} := d23_{i23,12}$

$pt2_2 := \mathrm{linterp}(alt2, pt2, 100)$ $t2_2 := \mathrm{linterp}(alt2, t2, 100)$ $p2_2 := \mathrm{linterp}(alt2, p2, 100)$ $rh2_2 := \mathrm{linterp}(alt2, rh2, 100)$

$dt2_2 := \mathrm{linterp}(alt2, dt2, 100)$

Data set K-2 with interpolated values at 100m:

$$d2 := \begin{bmatrix} 1 & 16 & 0.0 & 0.0 & 180 & 2.6 & 5.0 & 28.4 & 28.3 & 25.4 & 1011.0 & 84.0 \\ 2 & 0 & 0 & 100 & 224 & 6.2 & 12 & t2_2 & pt2_2 & dt2_2 & p2_2 & rh2_2 \\ 4 & 1000 & 984.0 & 299.9 & 224 & 6.2 & 12.0 & 28.7 & 28.9 & 21.6 & 977.7 & 66.0 \end{bmatrix}$$

d2 =

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	16	0	0	180	2.6	5	28.4	28.3	25.4	1011	84
2	2	0	0	100	224	6.2	12	28.5	28.5	24.1	999.9	78
3	4	1000	984	299.9	224	6.2	12	28.7	28.9	21.6	977.7	66

Note: Non-zero values of d2 at 100m interpolated

$i2 := 1 .. \mathrm{rows}(d2)$ $wd2_{i2} := d2_{i2,5}$ $pt2_{i2} := d2_{i2,9}$

$ML2_{i2} := d2_{i2,1}$ $ws2_{i2} := d2_{i2,6}$ $p2_{i2} := d2_{i2,11}$

$alt2_{i2} := d2_{i2,4}$ $t2_{i2} := \left(d2_{i2,8} + 273\right)$ $rh2_{i2} := d2_{i2,12}$

Potential Temperature Lapse Rate Calculations:

$j16 := 2 .. \mathrm{rows}(d16) - 1$ $j19 := 2 .. \mathrm{rows}(d19) - 1$ $j2 := 2 .. \mathrm{rows}(d2) - 1$

$$d\Theta dz16_{j16} := \frac{pt16_{j16+1} - pt16_{j16}}{alt16_{j16+1} - alt16_{j16}} \qquad d\Theta dz19_{j19} := \frac{pt19_{j19+1} - pt19_{j19}}{alt19_{j19+1} - alt19_{j19}} \qquad d\Theta dz2_{j2} := \frac{pt2_{j2+1} - pt2_{j2}}{alt2_{j2+1} - alt2_{j2}}$$

$j21 := 2 .. \mathrm{rows}(d21) - 1$ $j23 := 2 .. \mathrm{rows}(d23) - 1$

$$d\Theta dz21_{j21} := \frac{pt21_{j21+1} - pt21_{j21}}{alt21_{j21+1} - alt21_{j21}} \qquad d\Theta dz23_{j23} := \frac{pt23_{j23+1} - pt23_{j23}}{alt23_{j23+1} - alt23_{j23}}$$

$sf2 := \mathrm{mean}(d\Theta dz2)\cdot\left(alt2_{\mathrm{rows}(d2)} - alt2_2\right)$ $sf2 = 0.2$

$sf16 := \mathrm{mean}(d\Theta dz16)\cdot\left(alt16_{\mathrm{rows}(d16)} - alt16_2\right)$ $sf16 = -0.058$

$sf19 := \mathrm{mean}(d\Theta dz19)\cdot\left(alt19_{\mathrm{rows}(d19)} - alt19_2\right)$ $sf19 = 1.416$

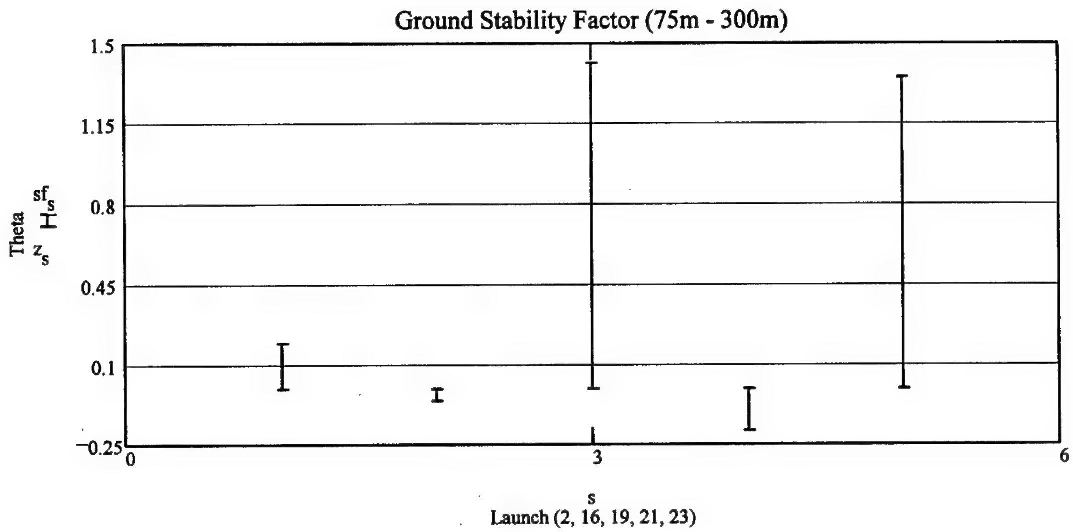
$sf21 := \mathrm{mean}(d\Theta dz21)\cdot\left(alt21_{\mathrm{rows}(d21)} - alt21_2\right)$ $sf21 = -0.188$

$sf23 := \mathrm{mean}(d\Theta dz23)\cdot\left(alt23_{\mathrm{rows}(d23)} - alt23_2\right)$ $sf23 = 1.355$

$sft := (sf2 \;\; sf16 \;\; sf19 \;\; sf21 \;\; sf23)$ $\quad sf := sft^T$ $\qquad zt := (0 \;\; 0 \;\; 0 \;\; 0 \;\; 0)$

$sft = (\,0.2 \;\; -0.058 \;\; 1.416 \;\; -0.188 \;\; 1.355\,)$ $\qquad z := zt^T$

$s := 1 .. 5$



$sft = (\,0.2 \;\; -0.06 \;\; 1.42 \;\; -0.19 \;\; 1.36\,)$

APPENDIX F

Regression equation determinations for coefficient of entrainment

UNWEIGHTED LEAST SQUARES LINEAR REGRESSION OF GAMMA

PREDICTOR VARIABLES	COEFFICIENT	STD ERROR	STUDENT'S T	P	VIF
CONSTANT	1.05257	0.44844	2.35	0.2564	
WINDSPD	0.03142	0.02790	1.13	0.4622	1.0
DEWPT	-0.04319	0.02334	-1.85	0.3154	1.9
STABFAC	-0.03917	0.08174	-0.48	0.7155	1.9

R-SQUARED	0.9139	RESID. MEAN SQUARE (MSE)	0.00988
ADJUSTED R-SQUARED	0.6558	STANDARD DEVIATION	0.09940

SOURCE	DF	SS	MS	F	P
REGRESSION	3	0.10492	0.03497	3.54	0.3681
RESIDUAL	1	0.00988	0.00988		
TOTAL	4	0.11480			

CASES INCLUDED 5 MISSING CASES 1

UNWEIGHTED LEAST SQUARES LINEAR REGRESSION OF GAMMA

PREDICTOR VARIABLES	COEFFICIENT	STD ERROR	STUDENT'S T	P	VIF
CONSTANT	1.21823	0.34425	3.54	0.1753	
WINDSPD	0.04196	0.03298	1.27	0.4241	1.6
DEWPT	-0.05625	0.01826	-3.08	0.1998	1.3
HIGHLR	0.03851	0.06461	0.60	0.6578	1.9

R-SQUARED	0.9219	RESID. MEAN SQUARE (MSE)	0.00896
ADJUSTED R-SQUARED	0.6877	STANDARD DEVIATION	0.09468

SOURCE	DF	SS	MS	F	P
REGRESSION	3	0.10584	0.03528	3.94	0.3511
RESIDUAL	1	0.00896	0.00896		
TOTAL	4	0.11480			

CASES INCLUDED 5 MISSING CASES 1

APPENDIX G

Mean square difference calculations

Stabilization Height for Various γ predictors and Mean Square Difference Calculations:

ORIGIN ≡ 1

$\gamma t := (.24 \;\; .30 \;\; .32 \;\; .42 \;\; .67)$ $\gamma := \gamma t^T$

$j := 1..5$

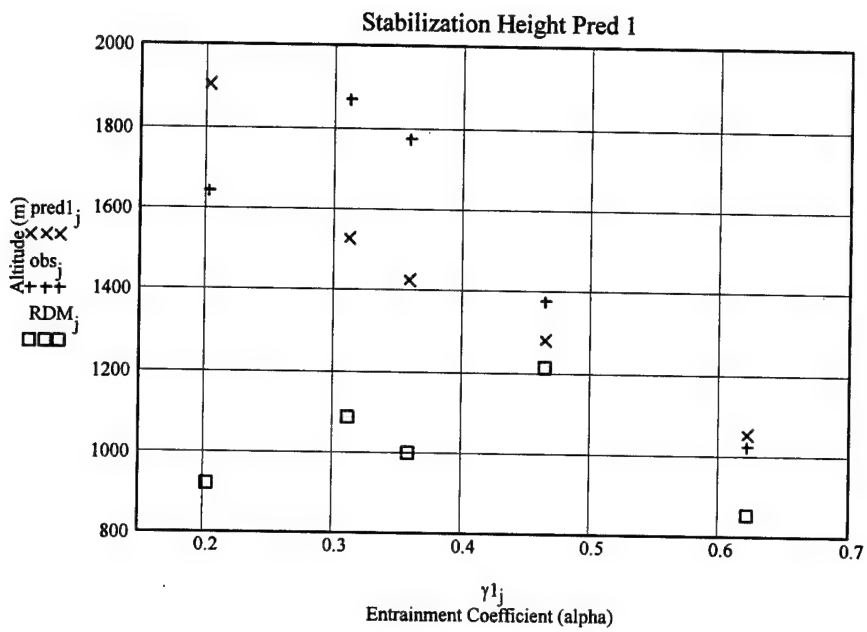
Observed Heights:

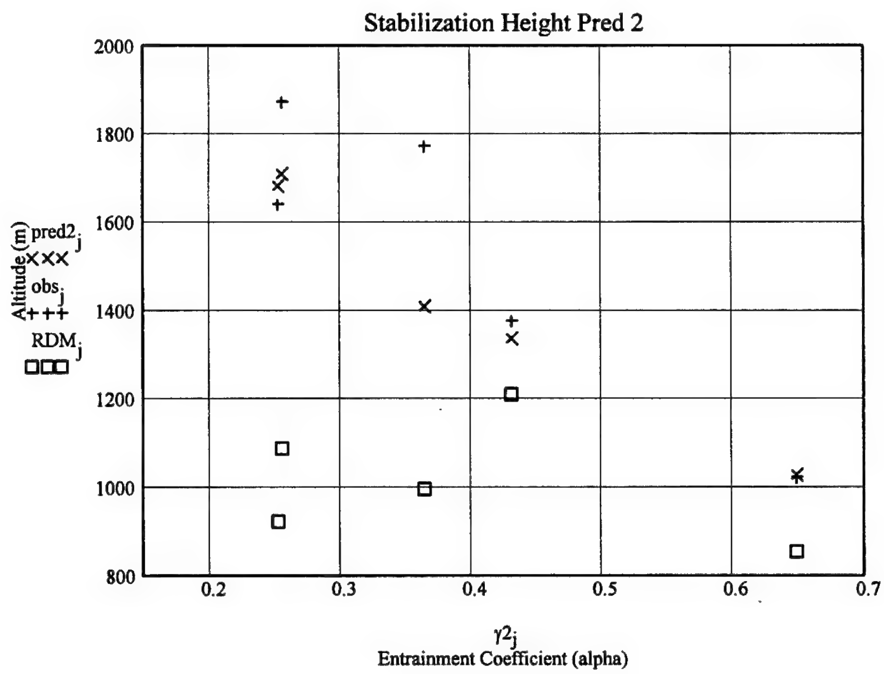
$obst := (1640 \;\; 1774 \;\; 1871 \;\; 1375 \;\; 1023)$ $obs := obst^T$

REEDM Predicted Heights:

$RDMt := (923 \;\; 997 \;\; 1087 \;\; 1211 \;\; 854)$ $RDM := RDMt^T$

$$pred1 := \begin{bmatrix} 1904 \\ 1424 \\ 1526 \\ 1280 \\ 1051 \end{bmatrix} \quad \gamma 1 := \begin{bmatrix} .203 \\ .358 \\ .312 \\ .465 \\ .622 \end{bmatrix} \quad pred2 := \begin{bmatrix} 1682 \\ 1410 \\ 1710 \\ 1338 \\ 1027 \end{bmatrix} \quad \gamma 2 := \begin{bmatrix} .252 \\ .364 \\ .256 \\ .431 \\ .648 \end{bmatrix}$$





Predictor Equations:

$$\gamma_{pred1} = 1.02319 - .03223 \cdot sf - .04244 \cdot dp + .03019 \cdot ws$$

$$\gamma_{pred3} = 1.16548 - .03829 \cdot hsf - .05409 \cdot dp + .04087 \cdot ws$$

Where:

- **sf = low level stability factor**
- **dp = low level dew point**
- **ws = low level wind speed**
- **hsf = high level stab factor**

<u>Observed γ</u> **<u>Prediction 1 γ</u>** **<u>Prediction 2 γ</u>**

$$\gamma = \begin{bmatrix} 0.24 \\ 0.3 \\ 0.32 \\ 0.42 \\ 0.67 \end{bmatrix} \qquad \gamma1 = \begin{bmatrix} 0.203 \\ 0.358 \\ 0.312 \\ 0.465 \\ 0.622 \end{bmatrix} \qquad \gamma2 = \begin{bmatrix} 0.252 \\ 0.364 \\ 0.256 \\ 0.431 \\ 0.648 \end{bmatrix} \qquad \gamma_{rdm} := \begin{bmatrix} .64 \\ .64 \\ .64 \\ .64 \\ .64 \end{bmatrix}$$

Mean Square Differences for the Different γ Predictors:

$$\Delta\gamma1_j := \left(\gamma_j - \gamma1_j\right)^2 \qquad \Delta\gamma2_j := \left(\gamma_j - \gamma2_j\right)^2 \qquad \Delta\gamma rdm_j := \left(\gamma_j - \gamma_{rdm_j}\right)^2$$

$$msd\gamma1 := \frac{\sqrt{\sum \Delta\gamma1}}{rows(\gamma)} \qquad msd\gamma2 := \frac{\sqrt{\sum \Delta\gamma2}}{rows(\gamma)} \qquad msd\gamma rdm := \frac{\sqrt{\sum \Delta\gamma rdm}}{rows(\gamma)}$$

$$msd\gamma1 = 0.019 \qquad msd\gamma2 = 0.02 \qquad msd\gamma rdm = 0.131$$

Mean Square Differences for the Different Predicted Stabilization Heights:

$\Delta h1_j := \left(obs_j - pred1_j\right)^2$ $\Delta h2_j := \left(obs_j - pred2_j\right)^2$ $\Delta hRDM_j := \left(obs_j - RDM_j\right)^2$

$msdh1 := \frac{\sqrt{\sum \Delta h1}}{rows(\gamma)}$ $msdh2 := \frac{\sqrt{\sum \Delta h2}}{rows(\gamma)}$ $msdh_{RDM} := \frac{\sqrt{\sum \Delta hRDM}}{rows(\gamma)}$

$msdh1 = 113.3$ $msdh2 = 80.4$ $msdh_{RDM} = 267.4$

Meteorological Data K-15:

ORIGIN ≡ 1

d15 :=

1	368	0.0	0.0	0	4.1	8.0	17.8	19.0	11.2	1002.9	65.0
2	422	54.0	16.5	13	5.1	10.0	16.9	18.3	11.0	1001.0	68.0
3	458	89.5	27.3	10	4.9	9.6	16.4	17.8	10.9	999.7	70.1
4	493	125.0	38.1	7	4.7	9.2	15.8	17.4	10.8	998.5	72.0
5	533	164.5	50.1	3	4.9	9.6	15.7	17.4	11.0	997.0	73.6
6	572	204.0	62.2	358	5.1	10.0	15.6	17.4	11.2	995.6	74.8
7	620	252.0	76.8	350	5.9	11.5	15.5	17.5	11.4	993.9	76.8
8	668	300.0	91.4	341	6.7	13.0	15.3	17.5	11.6	992.2	78.3
9	716	348.0	106.1	346	8.3	16.1	15.2	17.5	11.8	990.5	80.0
10	821	453.0	138.1	346	10.8	20.9	17.9	20.8	13.4	986.8	75.0
11	923	554.5	169.0	350	10.3	20.0	18.0	21.2	13.1	983.2	72.9
12	1024	656.0	199.9	353	9.9	19.2	18.2	21.6	12.8	979.7	71.1
13	1188	820.0	249.9	0	8.4	16.3	18.5	22.4	12.4	974.0	67.9
14	1393	1025.0	312.4	17	8.5	16.6	18.8	23.2	11.8	967.0	63.9
15	1516	1148.0	349.9	25	10.2	19.8	18.7	23.5	11.7	962.8	63.6
16	1680	1312.0	399.9	17	7.3	14.2	18.6	23.9	11.5	957.2	63.1
17	1762	1394.0	424.9	29	7.8	15.2	18.6	24.1	11.4	954.4	63.0
18	1844	1476.0	449.9	41	8.4	16.3	18.6	24.4	11.3	951.7	62.6
19	1926	1558.0	474.9	26	7.0	13.7	18.5	24.6	11.2	948.9	62.4
20	2008	1640.0	499.9	11	5.7	11.1	18.5	24.8	11.1	946.1	62.2
21	2518	2150.0	655.3	26	8.9	17.3	18.2	26.0	10.5	929.2	60.7
22	3074	2705.0	824.6	13	8.4	16.4	17.5	26.8	8.8	911.0	57.1
23	3629	3261.0	994.0	360	8.0	15.5	16.7	27.6	7.1	893.1	53.2
24	4198	3830.0	1167.5	358	7.8	15.2	15.4	27.9	6.3	875.1	55.0
25	4768	4400.0	1341.1	356	7.7	14.9	14.0	28.2	5.4	857.4	55.9
26	5326	4958.0	1511.2	351	7.5	14.6	13.8	29.6	3.6	840.3	50.7
27	5884	5516.0	1681.3	346	7.4	14.3	13.6	31.1	1.8	823.5	44.6
28	6470	6102.0	1859.9	332	7.5	14.6	12.1	31.4	2.6	806.2	52.0
29	6927	6559.0	1999.2	319	7.4	14.4	11.7	32.1	-2.5	793.0	37.0
30	7889	7521.0	2292.4	307	6.4	12.5	9.9	33.2	-4.6	765.6	35.6
31	8405	8037.0	2449.7	298	5.9	11.4	9.3	34.2	-5.1	751.2	36.8
32	8921	8553.0	2607.0	290	5.3	10.3	8.6	35.1	-5.6	737.1	35.9
33	9481	9113.0	2777.6	285	5.1	9.8	7.2	35.4	-5.6	722.0	40.6
34	10041	9673.0	2498.3	280	4.8	9.4	5.8	35.7	-5.7	707.2	43.4

$rd := \mathrm{rows}(d15)$

$i := 1 .. rd$

$ML_i := d15_{i,1}$

$alt_i := d15_{i,4} \cdot m$

$wd_i := d15_{i,5}$

$ws_i := d15_{i,6} \cdot m \cdot sec^{-1}$

$t_i := d15_{i,8} \cdot K$

$pt_i := d15_{i,9} \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d15_{i,11} \cdot mb$

K-15 Plume Rise Model

Model parameters:

$\alpha := .919 \qquad g = 9.807 \cdot m \cdot sec^{-2} \qquad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1} \qquad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3} \qquad \rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 7$

Temp: $t_{meti} = 15.5 \cdot K$ **Pres:** $p_{meti} = 9.939 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}} \qquad \rho_{00} = 22.342 \cdot kg \cdot m^{-3} \qquad alt_{meti} = 76.8 \cdot m \qquad \rho_1 := \rho_{00}$

Final density: $metf := 21$

Temp: $t_{metf} = 18.2 \cdot K$ **Pres:** $p_{metf} = 9.292 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}} \qquad \rho_{0f} = 17.789 \cdot kg \cdot m^{-3} \qquad alt_{metf} = 655.3 \cdot m$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right) \qquad d\rho dz = -7.871 \cdot 10^{-3} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1} \qquad G = 3.455 \cdot 10^{-3} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3} \qquad v2u := sec \cdot kg^{-1} \cdot m^{-1} \qquad v3u := sec^2 \cdot m^{-4} \qquad v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0{}^3 \cdot v1u \qquad v_2 := r_0{}^3 \cdot \rho_i \cdot u_0 \cdot v2u \qquad v_3 := r_0{}^3 \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u \qquad v_4 := 0 \cdot m \cdot v4u$

$v^T = (\, 4.913 \cdot 10^6 \quad 0 \quad 4.818 \cdot 10^7 \quad 0 \,)$

Determination of critical values:

$$\mathrm{rhs}(t,v) := \left|\begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array}\right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $\quad n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := \mathrm{rkfixed}(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{\langle 2 \rangle} \cdot \dfrac{1}{v1u}$ $\quad rad_i := (v1_i)^{\frac{1}{3}}$ $\quad Hu := z^{\langle 5 \rangle} \cdot \dfrac{1}{v4u} + r_0$ $\quad Hu_m := \max(Hu)$

$\max(rad) = 563 \cdot m$ **<== Max plume radius**

$Hu_m = 691 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{\langle 3 \rangle} \cdot \dfrac{1}{v2u}$ $\quad v3 := z^{\langle 4 \rangle} \cdot \dfrac{1}{v3u}$

Observed Cloud Rise Data:

$t_{ob} := 11 \cdot 60$ $\quad j := 1 .. t_{ob} \cdot n + 1$ $\quad x_j := \dfrac{j}{60 \cdot n}$ $\quad (ft - it) \cdot n = 1 \cdot 10^3$ $\quad n = 10$

$zob_j := -0.5498 \cdot (x_j)^4 + 14.152 \cdot (x_j)^3 - 128.34 \cdot (x_j)^2 + 502.52 \cdot x_j - 66.388$ $\quad \max(zob) = 781.354$

$vob_j := \left[-2.1992 \cdot (x_j)^3 + 42.456 \cdot (x_j)^2 - 256.68 \cdot x_j + 502.52\right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $\quad u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot \sec \cdot m^{-1}$ $\quad \alpha = 0.919$

$H_i := Hu_i \cdot v4u$

Predictor Equations:

$\gamma_{pred1} = 1.02319 - .03223 \cdot sf - .04244 \cdot dp + .03019 \cdot ws$

$\gamma_{pred2} = .225 + .32355 + .05229 - .06543 \cdot sf - .01219 \cdot dp + .00719 \cdot ws$

$\gamma_{pred3} = 1.16548 - .03829 \cdot hsf - .05409 \cdot dp + .04087 \cdot ws$

Where:
- **sf = low level stability factor**
- **dp = low level dew point**
- **ws = low level wind speed**
- **hsf = high level stab factor**

$jh15 := 14..29 \quad ih15 := 1..29 - 13$

$d\Theta h_{ih15} := \left(pt_{jh15+1} - pt_{jh15}\right) \cdot K^{-1}$

$hsf := \text{mean}(d\Theta h) \qquad hsf = 1.1$

$jl15 := 7..14 \quad il15 := 1..14 - 6$

$d\Theta l_{il15} := \left(pt_{jl15+1} - pt_{jl15}\right) \cdot K^{-1}$

$sf := \text{mean}(d\Theta l) \qquad sf = 0.3$

$j15 := 7..21 \qquad idp15 := 1..21 - 6$

$dtl_{idp15} := d15_{j15,10}$

$dp := \text{mean}(dtl) \qquad dp = 10.5$

$j15 := 7..21 \qquad idp15 := 1..21 - 6$

$wsl_{idp15} := ws_{j15} \cdot m^{-1} \cdot sec$

$ws := \text{mean}(wsl) \qquad ws = 8.9$

$\gamma_{pred1} := 1.02319 - .03223 \cdot sf - .04244 \cdot dp + .03019 \cdot ws$

$\gamma_{pred2} := .225 + .32355 + .05229 - .06543 \cdot sf - .01219 \cdot dp + .00719 \cdot ws$

$\gamma_{pred3} := 1.16548 - .03829 \cdot hsf - .05409 \cdot dp + .04087 \cdot ws$

$\gamma_{pred1} = 0.837 \qquad \gamma_{pred2} = 0.517 \qquad \gamma_{pred3} = 0.919$

Meteorological Data K-22: ORIGIN $\equiv$ 1 **K-22**

$$
d22 := \begin{bmatrix}
1 & 329.0 & -39.0 & -11.9 & 355.0 & 3.1 & 6.0 & 19.4 & 19.4 & 10.7 & 1001.8 & 57.0 \\
2 & 383.0 & 15.0 & 4.6 & 0.0 & 3.1 & 6.0 & 18.7 & 18.7 & 9.9 & 999.9 & 56.0 \\
3 & 431.0 & 63.0 & 19.2 & 355.0 & 3.1 & 6.0 & 18.1 & 18.1 & 9.2 & 998.2 & 56.4 \\
4 & 513.0 & 145.0 & 44.2 & 354.3 & 3.1 & 6.0 & 17.0 & 17.0 & 8.0 & 995.3 & 56.0 \\
5 & 620.0 & 252.0 & 76.8 & 359.0 & 2.6 & 5.0 & 16.7 & 16.7 & 10.3 & 991.5 & 66.0 \\
6 & 829.0 & 461.0 & 140.5 & 345.0 & 3.4 & 6.6 & 15.0 & 15.1 & 9.4 & 984.1 & 69.0 \\
7 & 984.0 & 616.0 & 187.8 & 343.0 & 4.8 & 9.3 & 15.2 & 15.3 & 9.8 & 978.6 & 70.0 \\
8 & 1149.0 & 781.0 & 238.0 & 320.0 & 6.0 & 11.7 & 17.9 & 18.0 & 9.4 & 972.9 & 58.1 \\
9 & 1262.0 & 894.0 & 272.5 & 295.9 & 4.9 & 9.6 & 19.8 & 20.0 & 9.1 & 969.0 & 50.0 \\
10 & 1313.0 & 945.0 & 288.0 & 285.0 & 4.5 & 8.7 & 21.3 & 21.5 & 9.9 & 967.3 & 48.3 \\
11 & 1384.0 & 1016.0 & 309.7 & 289.3 & 5.0 & 9.7 & 23.4 & 23.6 & 11.1 & 964.8 & 46.0 \\
12 & 1477.0 & 1109.0 & 338.0 & 295.0 & 5.6 & 10.9 & 23.9 & 24.2 & 10.3 & 961.7 & 42.5 \\
13 & 1553.0 & 1185.0 & 361.2 & 323.4 & 5.9 & 11.4 & 24.3 & 24.6 & 9.7 & 959.2 & 39.6 \\
14 & 2059.0 & 1691.0 & 515.4 & 328.7 & 6.7 & 13.1 & 26.5 & 27.0 & 6.5 & 942.5 & 28.0 \\
15 & 2700.0 & 2332.0 & 710.8 & 333.4 & 6.9 & 13.4 & 25.8 & 26.4 & 4.5 & 921.8 & 25.3 \\
16 & 3884.0 & 3516.0 & 1071.7 & 340.1 & 5.5 & 10.7 & 23.9 & 24.8 & 0.7 & 884.6 & 21.6 \\
17 & 5072.0 & 4704.0 & 1433.8 & 1.9 & 3.1 & 6.1 & 20.9 & 21.9 & -1.2 & 848.5 & 22.7 \\
18 & 6290.0 & 5922.0 & 1805.0 & 34.3 & 1.7 & 3.3 & 18.1 & 19.2 & -1.9 & 812.7 & 25.5 \\
19 & 7538.0 & 7170.0 & 2185.4 & 287.0 & 0.4 & 0.8 & 17.1 & 18.4 & -8.3 & 777.3 & 16.8 \\
20 & 8754.0 & 8386.0 & 2556.1 & 251.2 & 2.2 & 4.3 & 15.2 & 16.5 & -10.0 & 744.2 & 16.6 \\
21 & 10063.0 & 9695.0 & 2955.0 & 245.4 & 2.8 & 5.5 & 13.7 & 15.1 & -11.0 & 709.9 & 16.9
\end{bmatrix}
$$

$rd := \mathrm{rows}(d22)$

$i := 1 .. rd$

$ML_i := d22_{i,1}$ $\quad wd_i := d22_{i,5}$ $\quad pt_i := \left(d22_{i,9} + 273\right)\cdot K$

$alt_i := d22_{i,4}\cdot m$ $\quad ws_i := d22_{i,6}\cdot m\cdot sec^{-1}$ $\quad mb := 100\cdot kg\cdot m^{-1}\cdot sec^{-2}$

$t_i := \left(d22_{i,8} + 273\right)\cdot K$ $\quad p_i := d22_{i,11}\cdot mb$

K-22 Plume Rise Model

Model parameters:

$\alpha := 1.165$ $\quad g = 9.807 \cdot m \cdot sec^{-2}$ $\quad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $\quad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\quad \rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 5$ $\quad alt_{meti} = 76.8 \cdot m$

Temp: $t_{meti} = 289.7 \cdot K$ **Pres:** $p_{meti} = 9.915 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\quad \rho_{00} = 1.193 \cdot kg \cdot m^{-3}$ $\quad \rho_1 := \rho_{00}$

Final density: $metf := 14$ $\quad alt_{metf} = 515.4 \cdot m$

Temp: $t_{metf} = 299.5 \cdot K$ **Pres:** $p_{metf} = 9.425 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\quad \rho_{0f} = 1.096 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right)$ $\quad d\rho dz = -2.189 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $\quad G = 1.8 \cdot 10^{-3} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $\quad v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $\quad v3u := sec^2 \cdot m^{-4}$ $\quad v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0{}^3 \cdot v1u$ $\quad v_2 := r_0{}^3 \cdot \rho_i \cdot u_0 \cdot v2u$ $\quad v_3 := r_0{}^3 \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u$ $\quad v_4 := 0 \cdot m \cdot v4u$

$v^T = (4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0)$

Determination of critical values:

$$\mathrm{rhs}(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow \left(v_1\right)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $\quad n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := \mathrm{rkfixed}(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{\langle 2 \rangle} \cdot \dfrac{1}{v1u}$ $\quad rad_i := \left(v1_i\right)^{\frac{1}{3}}$ $\quad Hu := z^{\langle 5 \rangle} \cdot \dfrac{1}{v4u} + r_0$ $\quad Hu_m := \max(Hu)$

$\max(rad) = 704 \cdot m$ **<== Max plume radius**

$Hu_m = 630 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{\langle 3 \rangle} \cdot \dfrac{1}{v2u}$ $\quad v3 := z^{\langle 4 \rangle} \cdot \dfrac{1}{v3u}$

<u>Observed Cloud Rise Data:</u>

$t_{ob} := 11 \cdot 60$ $\quad j := 1 .. t_{ob} \cdot n + 1$ $\quad x_j := \dfrac{j}{60 \cdot n}$ $\quad (ft - it) \cdot n = 1 \cdot 10^3$ $\quad n = 10$

$zob_j := .0016 \cdot \left(x_j\right)^5 - .1495 \cdot \left(x_j\right)^4 + 4.9361 \cdot \left(x_j\right)^3 - 71.141 \cdot \left(x_j\right)^2 + 437.92 \cdot x_j - 229.17$ $\quad \max(zob) = 722.209$

$vob_j := \left[8.0 \cdot 10^{-3} \cdot \left(x_j\right)^4 - .598 \cdot \left(x_j\right)^3 + 14.8083 \cdot \left(x_j\right)^2 - 142.282 \cdot x_j + 437.92 \right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $\quad u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\quad \alpha = 1.165$

$H_i := Hu_i \cdot v4u$

Predictor Equations:

$\gamma_{pred1} = 1.02319 - .03223 \cdot sf - .04244 \cdot dp + .03019 \cdot ws$

$\gamma_{pred2} = .225 + .32355 + .05229 - .06543 \cdot sf - .01219 \cdot dp + .00719 \cdot ws$

$\gamma_{pred3} = 1.16548 - .03829 \cdot hsf - .05409 \cdot dp + .04087 \cdot ws$

Where:

- **sf = low level stability factor**
- **dp = low level dew point**
- **ws = low level wind speed**
- **hsf = high level stab factor**

$jh22 := 11..18 \quad ih22 := 1..18 - 10$

$d\Theta h_{ih22} := \left(pt_{jh22+1} - pt_{jh22}\right) \cdot K^{-1}$

$hsf := \text{mean}(d\Theta h) \quad hsf = -0.8$

$jl22 := 5..11 \quad il22 := 1..11 - 4$

$d\Theta_{il22} := \left(pt_{jl22+1} - pt_{jl22}\right) \cdot K^{-1}$

$sf := \text{mean}(d\Theta) \quad sf = 0.6$

$j22 := 5..14 \quad idp22 := 1..14 - 4$

$dtl_{idp22} := d22_{j22,10}$

$dp := \text{mean}(dtl) \quad dp = 6.5$

$j22 := 5..14 \quad idp22 := 1..14 - 4$

$wsl_{idp22} := ws_{j22} \cdot m^{-1} \cdot sec$

$ws := \text{mean}(wsl) \quad ws = 6.7$

$\gamma_{pred1} := 1.02319 - .03223 \cdot sf - .04244 \cdot dp + .03019 \cdot ws$

$\gamma_{pred2} := .225 + .32355 + .05229 - .06543 \cdot sf - .01219 \cdot dp + .00719 \cdot ws$

$\gamma_{pred3} := 1.16548 - .03829 \cdot hsf - .05409 \cdot dp + .04087 \cdot ws$

$\gamma_{pred1} = 0.93 \qquad \gamma_{pred2} = 0.531 \qquad \gamma_{pred3} = 1.118$

Stabilization Height as Function of Initial Cloud Radius (VAFB):

$\text{ORIGIN} \equiv 1$

$\gamma t := (.48 \;\; .52)$ $\qquad \gamma := \gamma t^T$

Observed Heights:

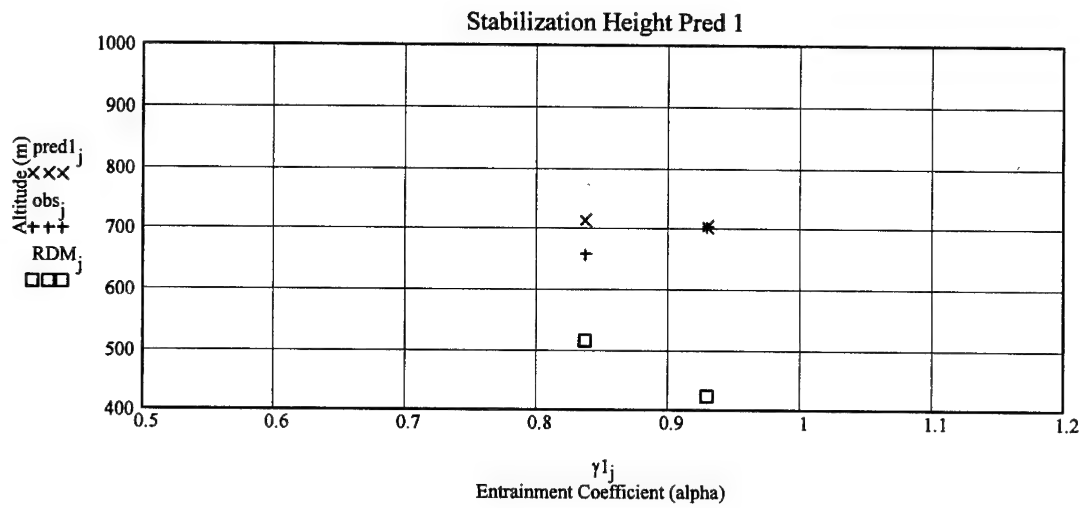
$ib := 1 .. \text{rows}(\gamma)$ $\qquad obst := (658 \;\; 702)$ $\qquad obs := obst^T$

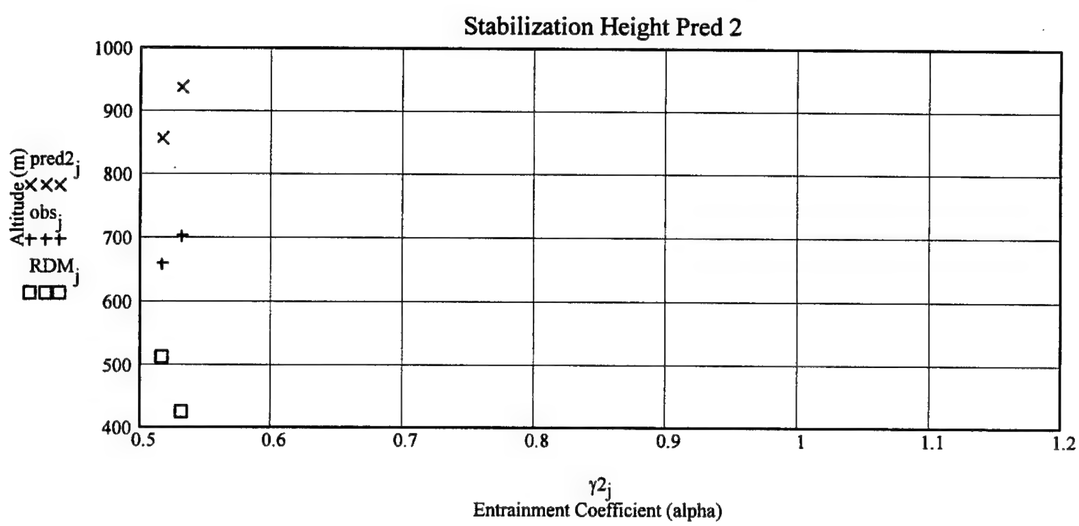
$j := 1 .. \text{rows}(\gamma)$

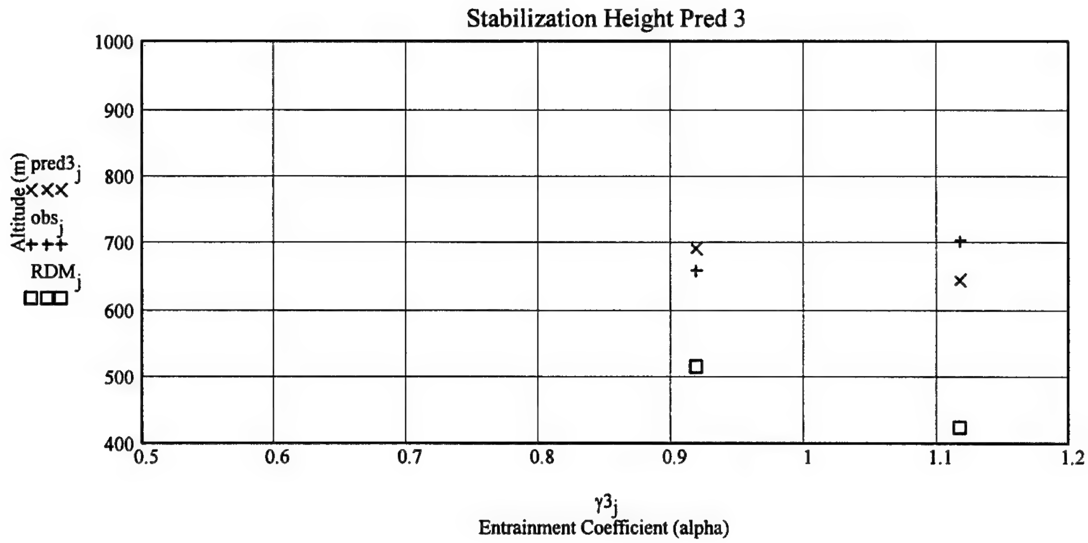
REEDM Predicted Heights:

$i := 2 .. \text{rows}(\gamma)$ $\qquad RDMt := (514 \;\; 424)$ $\qquad RDM := RDMt^T$

$pred1 := \begin{pmatrix} 714 \\ 704 \end{pmatrix}$ $\quad \gamma 1 := \begin{pmatrix} .837 \\ .93 \end{pmatrix}$ $\quad pred2 := \begin{pmatrix} 856 \\ 939 \end{pmatrix}$ $\quad \gamma 2 := \begin{pmatrix} .517 \\ .531 \end{pmatrix}$ $\quad pred3 := \begin{pmatrix} 691 \\ 643 \end{pmatrix}$ $\quad \gamma 3 := \begin{pmatrix} .919 \\ 1.118 \end{pmatrix}$







Predictor Equations:

$$\gamma_{pred1} = 1.02319 - .03223\cdot sf - .04244\cdot dp + .03019\cdot ws$$

$$\gamma_{pred2} = .225 + .32355 + .05229 - .06543\cdot sf - .01219\cdot dp + .00719\cdot ws$$

$$\gamma_{pred3} = 1.16548 - .03829\cdot hsf - .05409\cdot dp + .04087\cdot ws$$

Where:

- **sf = low level stability factor**
- **dp = low level dew point**
- **ws = low level wind speed**
- **hsf = high level stab factor**

Observed γ

$$\gamma = \begin{pmatrix} 0.48 \\ 0.52 \end{pmatrix}$$

Prediction 1 γ

$$\gamma1 = \begin{pmatrix} 0.837 \\ 0.93 \end{pmatrix}$$

Prediction 2 γ

$$\gamma2 = \begin{pmatrix} 0.517 \\ 0.531 \end{pmatrix}$$

Prediction 3 γ

$$\gamma3 = \begin{pmatrix} 0.919 \\ 1.118 \end{pmatrix}$$

Mean Square Differences for the Different γ Predictors:

$$\Delta\gamma1_j := \left(\gamma_j - \gamma1_j\right)^2 \qquad \Delta\gamma2_j := \left(\gamma_j - \gamma2_j\right)^2 \qquad \Delta\gamma3_j := \left(\gamma_j - \gamma3_j\right)^2$$

$$msd\gamma1 := \frac{\sqrt{\sum \Delta\gamma1}}{rows(\gamma)} \qquad msd\gamma2 := \frac{\sqrt{\sum \Delta\gamma2}}{rows(\gamma)} \qquad msd\gamma3 := \frac{\sqrt{\sum \Delta\gamma3}}{rows(\gamma)}$$

$$msd\gamma1 = 0.272 \qquad msd\gamma2 = 0.02 \qquad msd\gamma3 = 0.371$$

Mean Square Differences for the Different Predicted Stabilization Heights:

$$\Delta h1_j := \left(obs_j - pred1_j\right)^2 \qquad \Delta h2_j := \left(obs_j - pred2_j\right)^2 \qquad \Delta h3_j := \left(obs_j - pred3_j\right)^2 \qquad \Delta hRDM_j := \left(obs_j - RDM_j\right)^2$$

$$msdh1 := \frac{\sqrt{\sum \Delta h1}}{rows(\gamma)} \qquad msdh2 := \frac{\sqrt{\sum \Delta h2}}{rows(\gamma)} \qquad msdh3 := \frac{\sqrt{\sum \Delta h3}}{rows(\gamma)} \qquad msdh_{RDM} := \frac{\sqrt{\sum \Delta hRDM}}{rows(\gamma)}$$

$$msdh1 = 28 \qquad msdh2 = 154.4 \qquad msdh3 = 33.8 \qquad msdh_{RDM} = 156.5$$

APPENDIX H

Model templates

Meteorological Data K-2 (CCAS): ORIGIN ≡ 1 **K-2**

d2 :=

1	16	0.0	0.0	180	2.6	5.0	28.4	28.3	25.4	1011.0	84.0
2	300	284.0	86.6	193	3.6	7.0	28.49	28.5	24.3	1001.4	78.8
3	600	584.0	178.0	206	4.7	9.2	28.58	28.7	23.14	991.2	73.3
4	1000	984.0	299.9	224	6.2	12.0	28.7	28.9	21.6	977.7	66.0
5	1300	1284.0	391.4	226	6.2	12.1	28.07	28.3	21.0	967.8	65.7
6	1600	1584.0	482.8	229	6.3	12.2	27.44	27.8	20.4	958.0	65.4
7	2000	1984.0	604.7	232	6.4	12.4	26.6	27.0	19.6	944.8	65.0
8	2300	2284.0	696.2	234	6.3	12.2	25.73	26.2	19.21	935.2	67.1
9	2600	2584.0	787.6	236	6.2	12.0	24.86	25.4	18.82	925.6	69.2
10	3000	2984.0	909.5	238	6.1	11.8	23.7	24.3	18.3	912.8	72.0
11	3300	3284.0	1001.0	241	5.9	11.6	22.77	23.4	18.18	903.4	75.6
12	3600	3584.0	1092.4	244	5.8	11.3	21.84	22.5	18.06	894.0	79.2
13	4000	3984.0	1214.3	248	5.7	11.0	20.6	21.4	17.9	881.5	84.0
14	4300	4284.0	1305.8	252	5.6	10.9	19.85	20.6	17.3	872.3	84.9
15	4600	4584.0	1397.2	256	5.5	10.7	19.1	19.9	16.7	863.2	85.8
16	5000	4984.0	1519.1	261	5.4	10.5	18.1	19.0	15.9	851.0	87.0
17	5300	5284.0	1610.6	265	5.3	10.4	17.65	18.5	14.58	842.1	82.5
18	5600	5584.0	1702.0	269	5.3	10.3	17.2	18.1	13.26	833.2	78.0
19	6000	5984.0	1823.9	275	5.2	10.1	16.6	17.6	11.5	821.3	72.0
20	6300	6284.0	1915.4	278	5.1	9.9	16.3	17.3	9.4	812.6	64.8
21	6600	6584.0	2006.8	281	4.9	9.6	16.0	17.0	7.3	804.0	57.6
22	7000	6984.0	2128.7	285	4.8	9.3	15.6	16.7	4.5	792.5	48.0
23	8000	7984.0	2433.5	289	3.6	7.0	14.3	15.4	-0.5	764.5	36.0
24	9000	8984.0	2738.3	302	1.7	3.3	13.1	14.3	-4.8	737.4	28.0
25	10000	9984.0	3043.1	1	1.0	2.0	11.5	12.7	-3.8	711.1	35.0
26	11000	10984.0	3347.9	20	1.3	2.5	9.1	10.1	1.0	685.7	57.0
27	12000	11984.0	3652.7	20	1.3	2.5	7.3	8.2	-5.3	660.9	41.0

$rd := rows(d2)$

$i := 1 .. rd$

$ML_i := d2_{i,1}$

$alt_i := d2_{i,4} \cdot m$

$wd_i := d2_{i,5}$

$ws_i := d2_{i,6} \cdot m \cdot sec^{-1}$

$t_i := \left(d2_{i,8} + 273\right) \cdot K$

$pt_i := \left(d2_{i,9} + 273\right) \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d2_{i,11} \cdot mb$

K-2 Plume Rise Model

Model parameters:

$\alpha := .256$ $\quad g = 9.807 \cdot m \cdot sec^{-2}$ $\quad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $\quad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\quad \rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 2$ $\quad alt_{meti} = 86.6 \cdot m$

Temp: $t_{meti} = 301.49 \cdot K$ **Pres:** $p_{meti} = 1.001 \cdot 10^5 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\quad \rho_{00} = 1.157 \cdot kg \cdot m^{-3}$ $\quad \rho_1 := \rho_{00}$

Final density: $metf := 7$ $\quad alt_{metf} = 604.7 \cdot m$

Temp: $t_{metf} = 299.6 \cdot K$ **Pres:** $p_{metf} = 9.448 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\quad \rho_{0f} = 1.099 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right)$ $\quad d\rho dz = -1.13 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $\quad G = 9.572 \cdot 10^{-4} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $\quad v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $\quad v3u := sec^2 \cdot m^{-4}$ $\quad v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0^{\ 3} \cdot v1u$ $\quad v_2 := r_0^{\ 3} \cdot \rho_i \cdot u_0 \cdot v2u$ $\quad v_3 := r_0^{\ 3} \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u$ $\quad v_4 := 0 \cdot m \cdot v4u$

$v^T = (4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0)$

Determination of critical values:

$$rhs(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \frac{\rho_1}{g \cdot v_1} \\ u \leftarrow \frac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := rkfixed(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{\langle 2 \rangle} \cdot \frac{1}{v1u}$ $rad_i := (v1_i)^{\frac{1}{3}}$ $Hu := z^{\langle 5 \rangle} \cdot \frac{1}{v4u} + r_0$ $Hu_m := \max(Hu)$

$\max(rad) = 564 \cdot m$ **<== Max plume radius**

$Hu_m = 1710 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{\langle 3 \rangle} \cdot \frac{1}{v2u}$ $v3 := z^{\langle 4 \rangle} \cdot \frac{1}{v3u}$

<u>Observed Cloud Rise Data:</u>

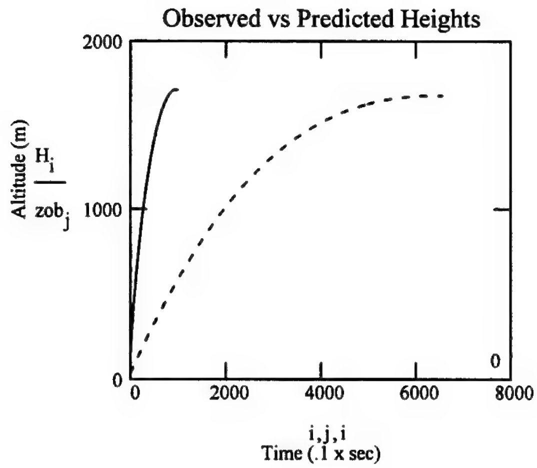
$t_{ob} := 11 \cdot 60$ $j := 1 .. t_{ob} \cdot n + 1$ $x_j := \frac{j}{60 \cdot n}$ $(ft - it) \cdot n = 1 \cdot 10^3$ $n = 10$

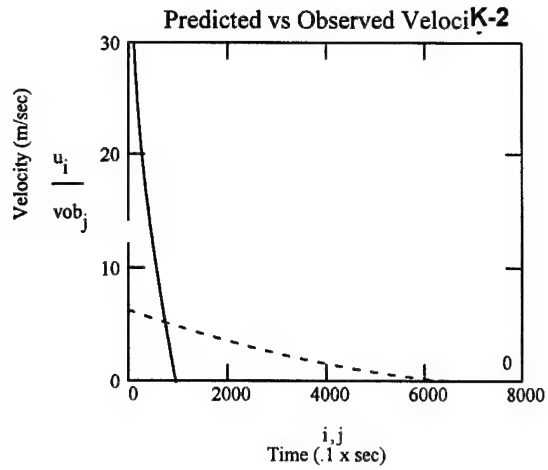
$zob_j := 0.6149 \cdot (x_j)^3 - 27.571 \cdot (x_j)^2 + 377.6 \cdot x_j + 38.08$ $\max(zob) = 1.675 \cdot 10^3$

$vob_j := \left[1.8447 \cdot (x_j)^2 - 55.142 \cdot x_j + 377.6 \right] \cdot \frac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \frac{\rho_1}{g \cdot v1_i}$ $u_i := \frac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\alpha = 0.256$

$H_i := Hu_i \cdot v4u$



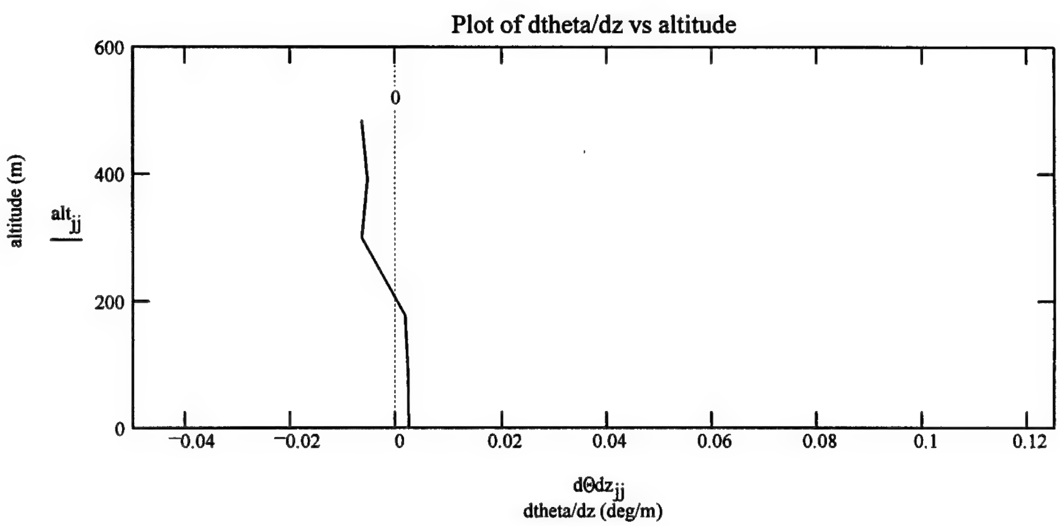


Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1 .. metf - 1$

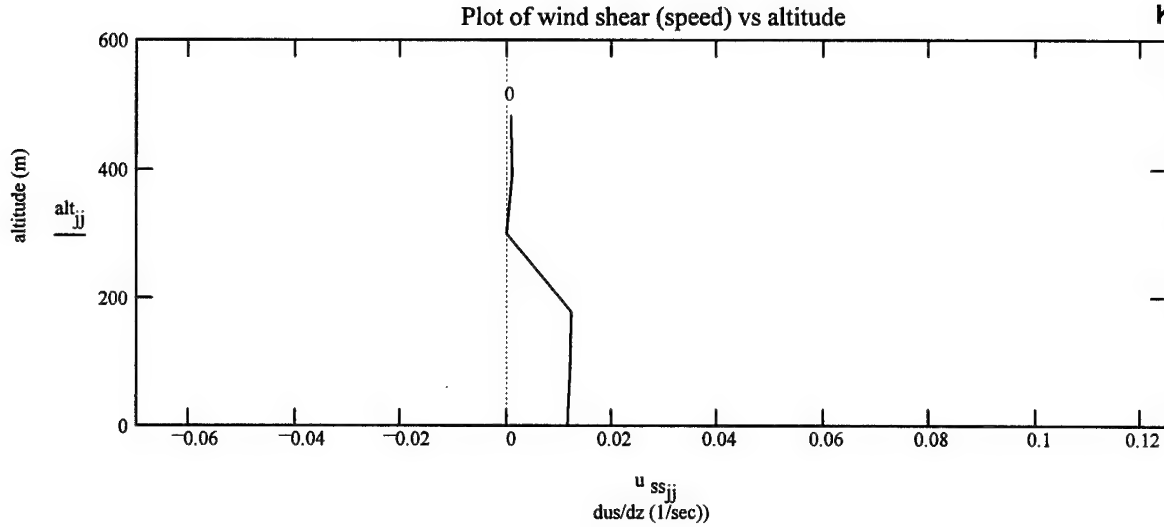
Determination of dtheta/dz:

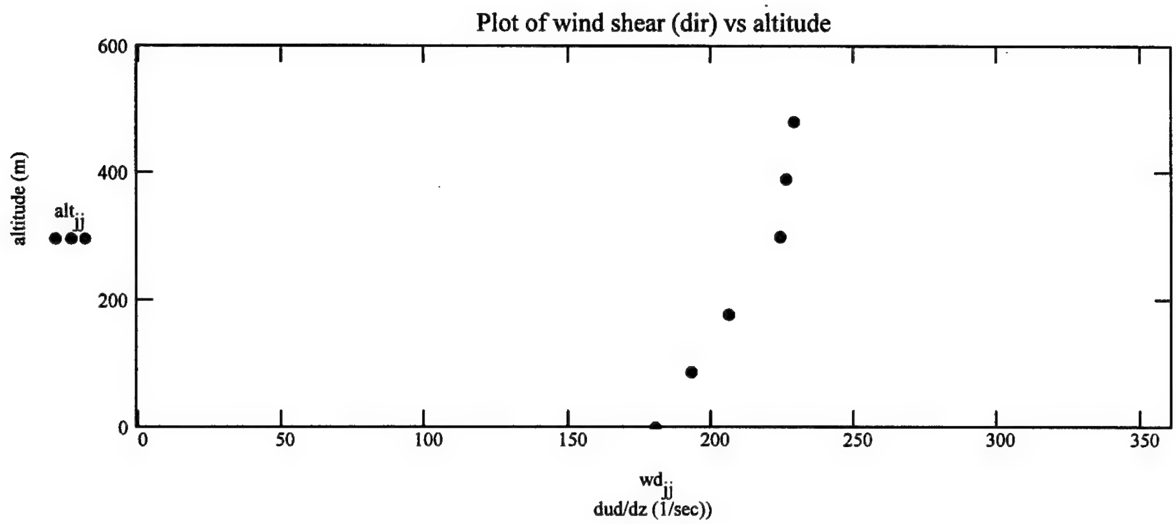
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}}\right) \cdot m \cdot K^{-1}$$



Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}} \cdot sec \qquad u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}} \cdot m \cdot deg^{-1}$$



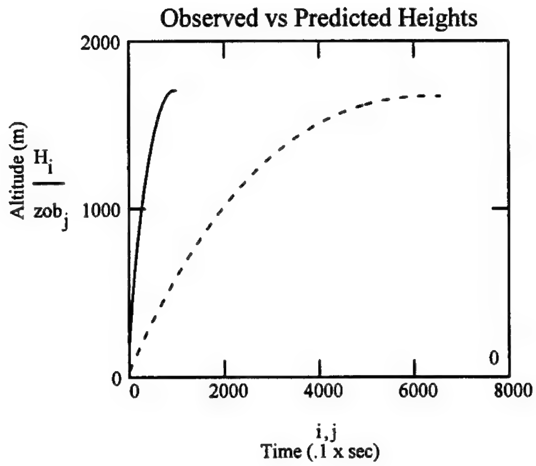


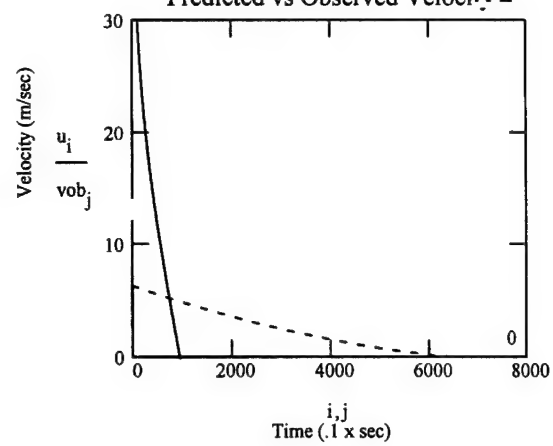
$ptj_{jj} := pt_{jj}$ $wsj_{jj} := ws_{jj}$ $wdj_{jj} := wd_{jj}$

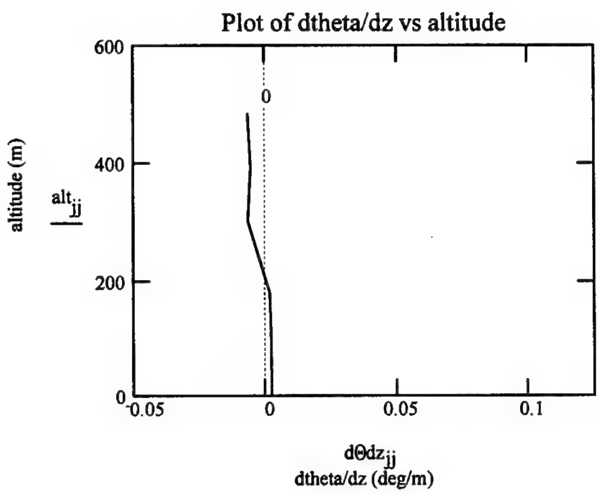
mean_pt := mean(ptj) var_pt := var(ptj)

mean_ws := mean(wsj) var_ws := var(wsj)

mean_wd := mean(wdj) var_wd := var(wdj)







mean_pt = 301.417•K

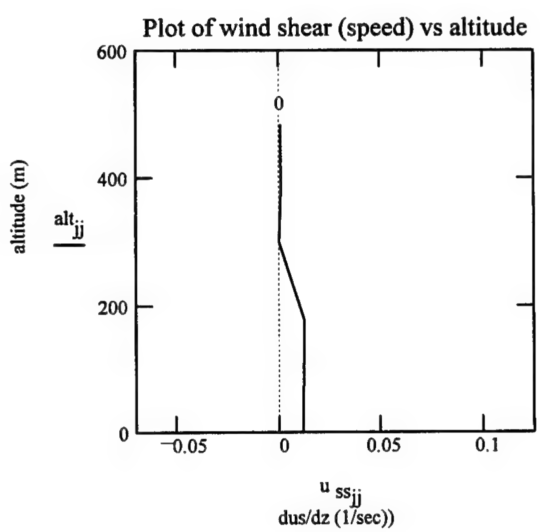
var_pt = 0.121•K^2

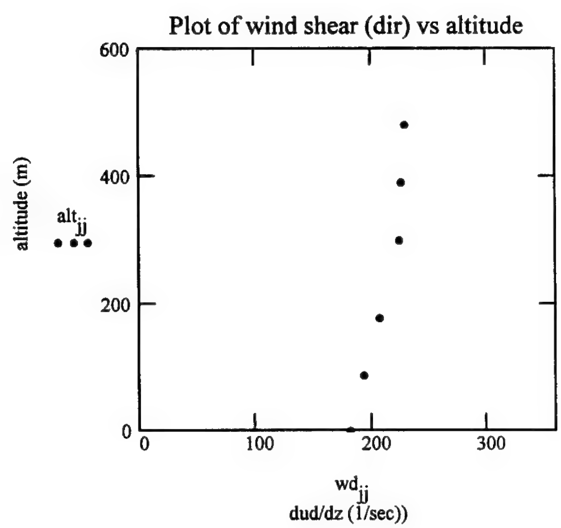
mean_ws = 4.933•m•sec^{-1}

var_ws = 2.059•m^2•sec^{-2}

mean_wd = 209.667

var_wd = 336.222





Meteorological Data K-16 (CCAS):

ORIGIN ≡ 1

d16 :=

1	16	0.0	0.0	20	6.2	12.0	20.8	21.3	16.3	1018.7	76.0
2	62	45.8	13.9	9	6.4	12.5	20.6	21.2	16.3	1017.1	76.4
3	108	91.5	27.9	359	6.7	13.0	20.4	21.1	16.3	1015.4	77.3
4	153	137.3	41.8	348	6.9	13.5	20.2	21.1	16.3	1013.8	78.3
5	199	183.0	55.8	337	7.2	14.0	20.0	21.0	16.3	1012.2	79.0
6	285	268.5	81.8	344	7.2	14.0	19.7	21.0	16.3	1009.1	80.9
7	370	354.0	107.9	351	7.2	14.0	19.5	21.0	16.4	1006.1	82.5
8	456	439.5	134.0	358	7.2	14.0	19.2	21.0	16.4	1003.0	84.2
9	541	525.0	160.0	5	7.2	14.0	18.9	21.0	16.5	1000.0	86.0
10	694	678.0	206.7	7	7.3	14.1	18.4	21.0	16.4	994.6	88.0
11	847	831.0	253.3	8	7.3	14.3	18.0	21.0	16.3	989.2	90.0
12	1000	984.0	299.9	10	7.4	14.4	17.5	20.9	16.2	983.9	92.0
13	1114	1098.0	334.7	11	7.2	14.0	17.2	21.0	16.1	979.9	93.0
14	1389	1372.5	418.3	11	7.2	14.0	17.1	21.5	14.3	970.4	83.6
15	1663	1647.0	502.0	10	7.2	14.0	17.0	22.0	12.5	961.0	75.0
16	2000	1984.0	604.7	4	6.5	12.6	16.2	22.4	13.8	949.5	86.0
17	2226	2210.0	673.6	359	6.2	12.0	15.7	22.7	14.8	941.9	94.0
18	2613	2597.0	791.6	346	6.1	11.9	15.8	24.0	14.6	929.0	93.2
19	3000	2984.0	909.5	332	6.1	11.9	15.8	25.2	14.5	916.2	91.0
20	3494	3478.0	1060.1	317	6.2	12.0	15.4	26.3	13.8	900.0	90.0
21	3862	3846.0	1172.3	304	6.2	12.0	15.0	27.0	13.4	888.4	90.0
22	4000	3984.0	1214.3	301	5.9	11.5	14.8	27.2	13.2	884.0	90.0
23	4500	4484.0	1366.7	290	6.2	12.0	13.9	27.7	12.3	868.3	90.2
24	5000	4984.0	1519.1	278	6.5	12.6	13.0	28.2	11.4	852.8	90.0
25	5082	5066.0	1544.1	278	6.7	13.0	12.9	28.4	11.3	850.0	90.0
26	5971	5955.0	1815.1	277	7.7	15.0	11.4	29.4	9.4	823.4	88.0

$rd := rows(d16)$

$i := 1 .. rd$

$ML_i := d16_{i,1}$

$alt_i := d16_{i,4} \cdot m$

$wd_i := d16_{i,5}$

$ws_i := d16_{i,6} \cdot m \cdot sec^{-1}$

$t_i := \left(d16_{i,8} + 273\right) \cdot K$

$pt_i := \left(d16_{i,9} + 273\right) \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d16_{i,11} \cdot mb$

K-16 Plume Rise Model

Model parameters:

$\alpha := .648 \qquad g = 9.807 \cdot m \cdot sec^{-2} \qquad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1} \qquad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.97349 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3} \qquad \rho_i = 1.931 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 6$

Temp: $t_{meti} = 292.7 \cdot K$ **Pres:** $p_{meti} = 1.009 \cdot 10^5 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}} \qquad \rho_{00} = 1.201 \cdot kg \cdot m^{-3} \qquad alt_{meti} = 81.8 \cdot m \qquad \rho_1 := \rho_{00}$

Final density: $metf := 16$

Temp: $t_{metf} = 289.2 \cdot K$ **Pres:** $p_{metf} = 9.495 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}} \qquad \rho_{0f} = 1.144 \cdot kg \cdot m^{-3} \qquad alt_{metf} = 604.7 \cdot m$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}}\right) \qquad d\rho dz = -1.095 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1} \qquad G = 8.941 \cdot 10^{-4} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3} \qquad v2u := sec \cdot kg^{-1} \cdot m^{-1} \qquad v3u := sec^2 \cdot m^{-4} \qquad v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0^{\,3} \cdot v1u \qquad v_2 := r_0^{\,3} \cdot \rho_i \cdot u_0 \cdot v2u \qquad v_3 := r_0^{\,3} \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1}\right) \cdot v3u \qquad v_4 := 0 \cdot m \cdot v4u$

$v^T = (\,4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0\,)$

Determination of critical values:

$$rhs(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow - G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := rkfixed(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{<2>} \cdot \dfrac{1}{v1u}$ $rad_i := (v1_i)^{\frac{1}{3}}$ $Hu := z^{<5>} \cdot \dfrac{1}{v4u} + r_0$ $Hu_m := max(Hu)$

$max(rad) = 725 \cdot m$ **<== Max plume radius**

$Hu_m = 1027 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{<3>} \cdot \dfrac{1}{v2u}$ $v3 := z^{<4>} \cdot \dfrac{1}{v3u}$

Observed Cloud Rise Data:

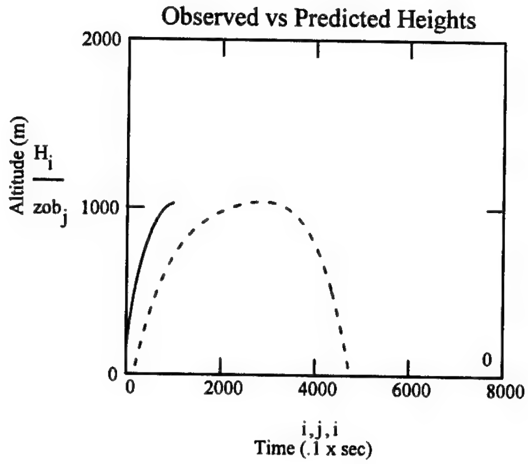
$t_{ob} := 11 \cdot 60$ $j := 1 .. t_{ob} \cdot n + 1$ $x_j := \dfrac{j}{60 \cdot n}$ $(ft - it) \cdot n = 1 \cdot 10^3$ $n = 10$

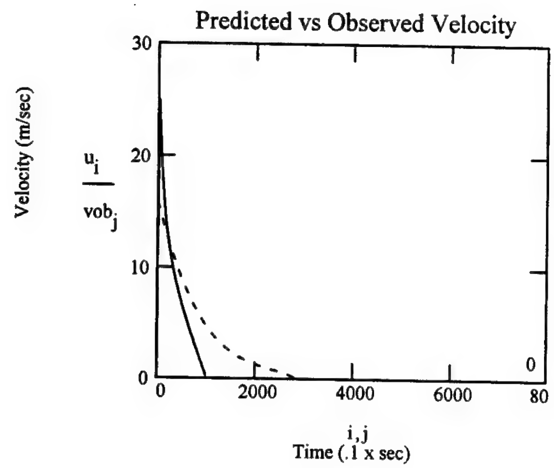
$zob_j := -3.1435 \cdot (x_j)^4 + 48.496 \cdot (x_j)^3 - 303.52 \cdot (x_j)^2 + 950.75 \cdot x_j - 225.75$ $max(zob) = 1.039 \cdot 10^3$

$vob_j := \left[-12.574 \cdot (x_j)^3 + 145.488 \cdot (x_j)^2 - 607.04 \cdot x_j + 950.75 \right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\alpha = 0.648$

$H_i := Hu_i \cdot v4u$



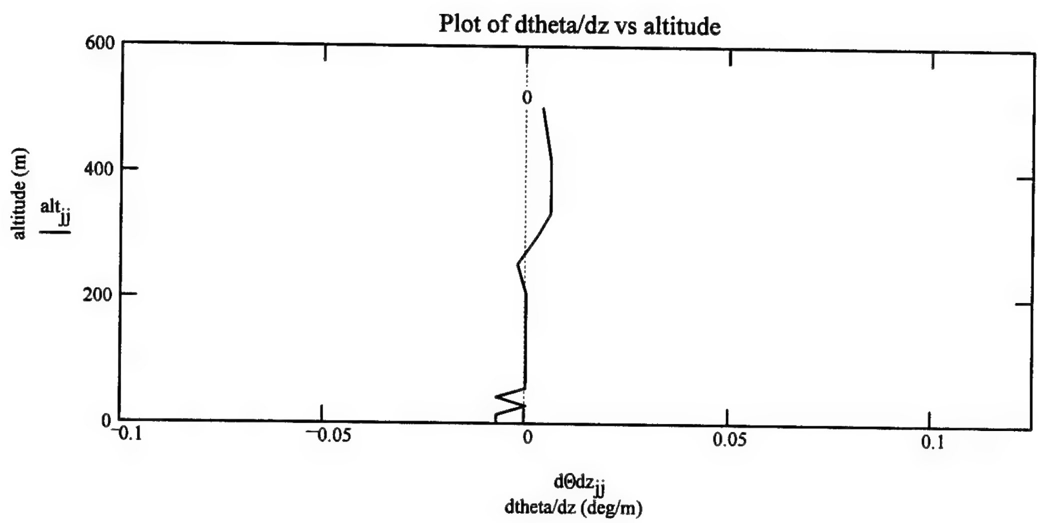


Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1 .. metf - 1$

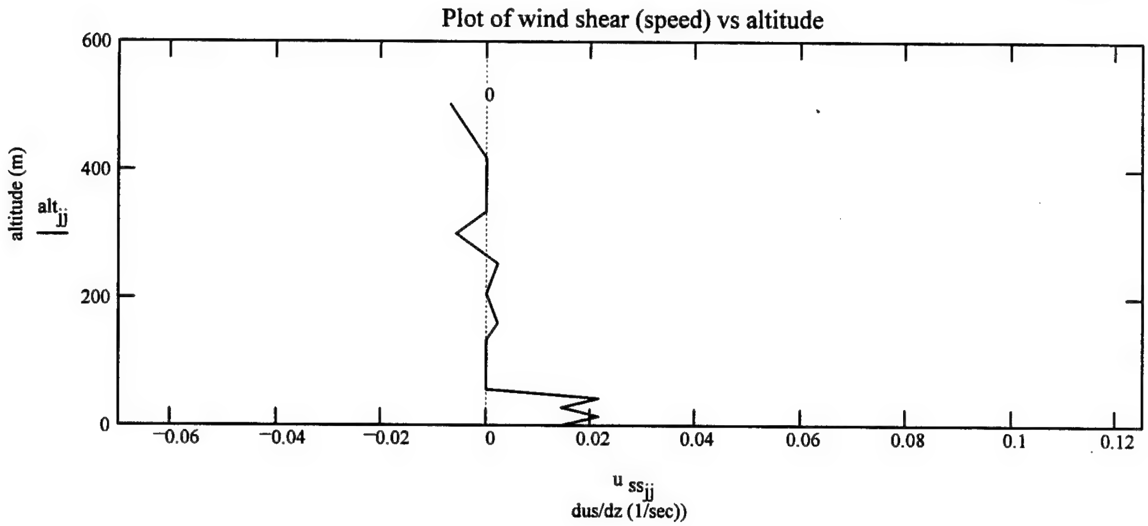
Determination of dtheta/dz:

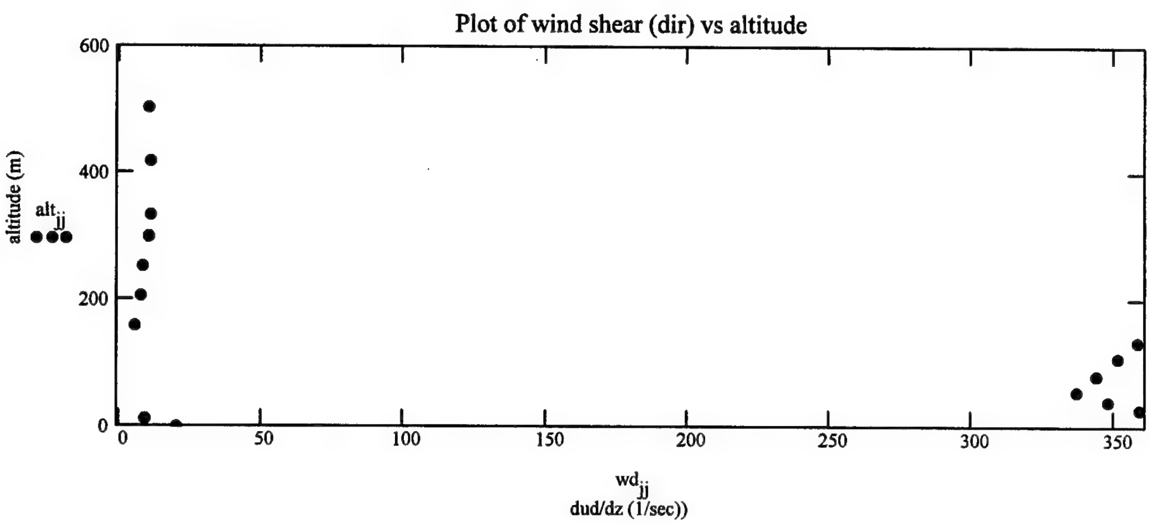
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}} \right) \cdot m \cdot K^{-1}$$



Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}} \cdot sec \qquad u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}} \cdot m \cdot deg^{-1}$$



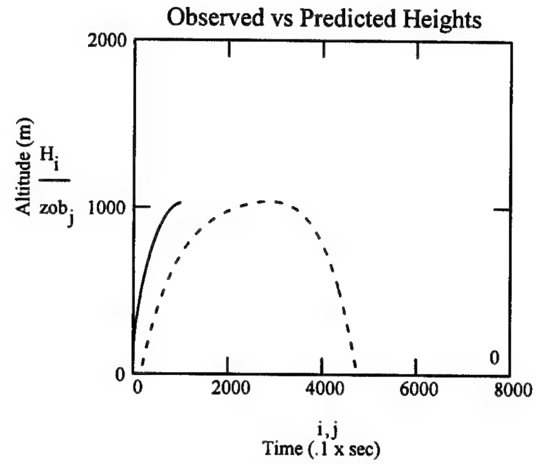


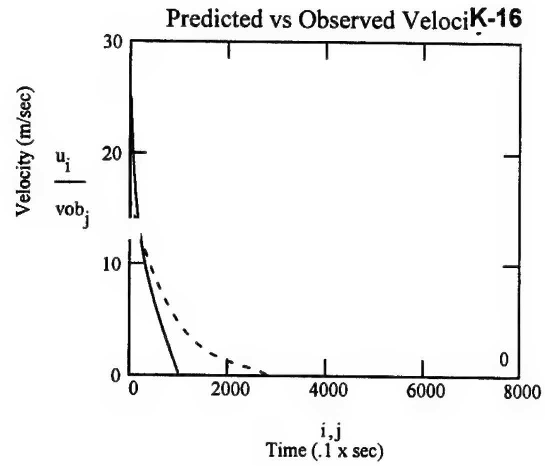
$ptj_{jj} := pt_{jj}$ $wsj_{jj} := ws_{jj}$ $wdj_{jj} := wd_{jj}$

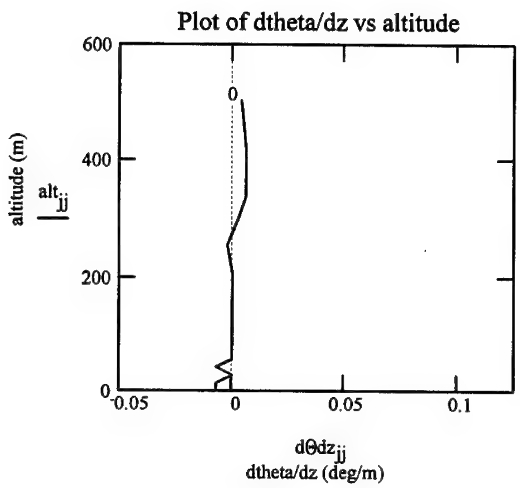
mean_pt := mean(ptj) var_pt := var(ptj)

mean_ws := mean(wsj) var_ws := var(wsj)

mean_wd := mean(wdj) var_wd := var(wdj)







$\text{mean\_pt} = 294.14 \cdot K$

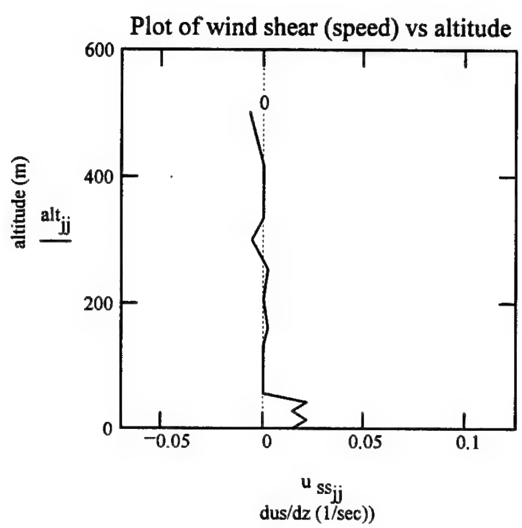
$\text{var\_pt} = 0.074 \cdot K^2$

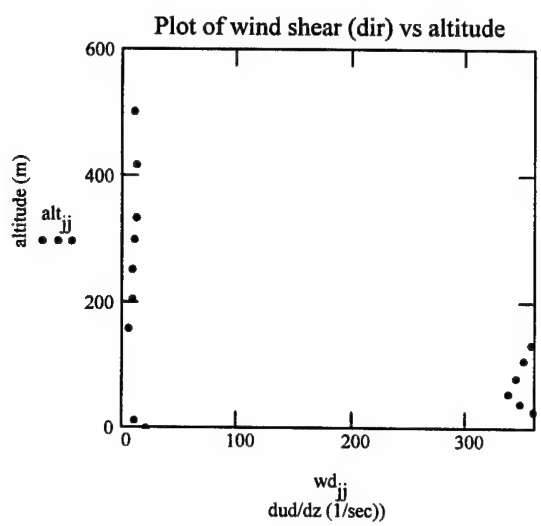
$\text{mean\_ws} = 7.053 \cdot m \cdot sec^{-1}$

$\text{var\_ws} = 0.114 \cdot m^2 \cdot sec^{-2}$

$\text{mean\_wd} = 145.867$

$\text{var\_wd} = 2.768 \cdot 10^4$





Meteorological Data K-19: ORIGIN ≡ 1

d19 :=

1	16	0.0	0.0	170	1.5	3.0	25.1	26.9	22.8	1014.8	87.0
2	57	40.6	12.4	187	2.6	5.0	25.3	27.3	22.9	1013.4	86.6
3	97	81.2	24.7	204	3.6	7.0	25.4	27.5	22.9	1012.0	86.1
4	138	121.8	37.1	221	4.6	9.0	25.6	27.8	23.0	1010.5	85.6
5	178	162.4	49.5	238	5.7	11.0	25.7	28.1	23.0	1009.1	85.1
6	219	203.0	61.9	255	6.7	13.0	25.9	28.5	23.1	1007.7	85.0
7	293	277.3	84.5	255	6.7	13.0	26.0	28.7	22.7	1005.1	81.9
8	368	351.7	107.2	255	6.7	13.0	26.1	28.9	22.2	1002.6	79.3
9	442	426.0	129.8	255	6.7	13.0	26.2	29.2	21.8	1000.0	77.0
10	533	517.0	157.6	255	6.9	13.5	26.4	29.5	21.3	996.9	73.6
11	624	608.0	185.3	255	7.2	14.0	26.5	29.9	20.7	993.8	71.0
12	812	796.0	242.6	255	7.4	14.3	26.0	29.9	20.4	987.4	71.1
13	1000	984.0	299.9	254	7.5	14.6	25.6	30.0	20.1	981.0	72.0
14	1309	1292.7	394.0	251	7.8	15.1	24.9	30.2	19.3	970.6	71.2
15	1617	1601.3	488.1	249	8.0	15.5	24.3	30.3	18.6	960.2	70.6
16	1926	1910.0	582.2	246	8.2	16.0	23.6	30.5	17.8	950.0	70.0
17	2000	1984.0	604.7	245	8.2	16.0	23.5	30.6	17.5	947.7	69.0
18	2500	2484.0	757.1	239	8.7	17.0	22.5	31.0	16.3	931.3	68.0
19	3000	2984.0	909.5	232	9.3	18.0	21.6	31.4	15.1	915.2	66.0
20	3472	3456.0	1053.4	228	9.8	19.0	20.8	32.0	14.8	900.0	68.0
21	4000	3984.0	1214.3	224	10.0	19.5	19.8	32.5	13.9	883.6	69.0
22	5000	4984.0	1519.1	218	10.3	20.0	17.7	33.4	13.3	852.9	76.0
23	5088	5072.0	1545.9	218	10.3	20.0	17.4	33.4	13.1	850.0	76.0
24	5602	5586.0	1702.6	216	10.3	20.0	16.4	33.8	11.9	834.9	75.0
25	6000	5984.0	1823.9	215	10.4	20.2	15.8	34.3	10.8	823.1	73.0
26	6153	6137.0	1870.6	214	10.3	20.0	15.5	34.4	10.4	818.6	72.0
27	6782	6766.0	2062.3	214	10.3	20.0	14.1	34.7	7.4	800.0	64.0
28	7000	6984.0	2128.7	214	10.1	19.7	13.7	34.8	6.4	794.1	61.0
29	7500	7484.0	2281.1	215	9.7	18.9	13.0	35.4	2.6	779.9	49.7
30	8000	7984.0	2433.5	216	9.3	18.1	12.4	36.1	-1.3	765.9	40.0
31	8400	8384.0	2555.4	217	8.7	17.0	12.4	37.0	-6.9	754.9	25.0
32	8561	8545.0	2604.5	217	8.7	17.0	12.1	37.3	-8.0	750.0	24.0
33	9000	8984.0	2738.3	218	8.2	15.9	11.3	37.7	-10.5	738.6	21.0
34	9500	9484.0	2890.7	220	7.9	15.4	10.4	38.3	-11.3	725.2	21.5

$rd := \mathrm{rows}(d19)$

$i := 1..rd$

$ML_i := d19_{i,1}$

$alt_i := d19_{i,4} \cdot m$

$wd_i := d19_{i,5}$

$ws_i := d19_{i,6} \cdot m \cdot sec^{-1}$

$t_i := \left(d19_{i,8} + 273\right) \cdot K$

$pt_i := \left(d19_{i,9} + 273\right) \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d19_{i,11} \cdot mb$

K-19 Plume Rise Model

Model parameters:

$\alpha := .364$ $\quad g = 9.807 \cdot m \cdot sec^{-2}$ $\quad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $\quad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\quad \rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 7$ $\quad alt_{meti} = 84.5 \cdot m$

Temp: $t_{meti} = 299 \cdot K$ **Pres:** $p_{meti} = 1.005 \cdot 10^5 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\quad \rho_{00} = 1.171 \cdot kg \cdot m^{-3}$ $\quad \rho_1 := \rho_{00}$

Final density: $metf := 17$ $\quad alt_{metf} = 604.7 \cdot m$

Temp: $t_{metf} = 296.5 \cdot K$ **Pres:** $p_{metf} = 9.477 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\quad \rho_{0f} = 1.114 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right)$ $\quad d\rho dz = -1.107 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $\quad G = 9.267 \cdot 10^{-4} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $\quad v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $\quad v3u := sec^2 \cdot m^{-4}$ $\quad v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0^{\,3} \cdot v1u$ $\quad v_2 := r_0^{\,3} \cdot \rho_i \cdot u_0 \cdot v2u$ $\quad v_3 := r_0^{\,3} \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u$ $\quad v_4 := 0 \cdot m \cdot v4u$

$v^T = (4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0)$

Determination of critical values:

$$\mathrm{rhs}(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $\quad n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := \mathrm{rkfixed}(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{<2>} \cdot \dfrac{1}{v1u}$ $\quad rad_i := (v1_i)^{\frac{1}{3}}$ $\quad Hu := z^{<5>} \cdot \dfrac{1}{v4u} + r_0$ $\quad Hu_m := \max(Hu)$

$\max(rad) = 621 \cdot m$ <== **Max plume radius**

$Hu_m = 1410 \cdot m$ <== **Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{<3>} \cdot \dfrac{1}{v2u}$ $\quad v3 := z^{<4>} \cdot \dfrac{1}{v3u}$

<u>Observed Cloud Rise Data:</u>

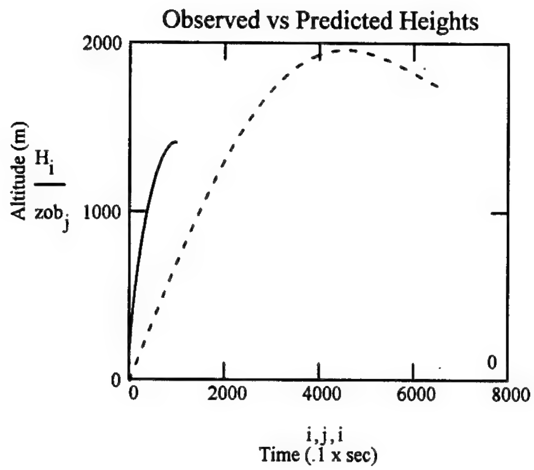
$t_{ob} := 11 \cdot 60$ $\quad j := 1 .. t_{ob} \cdot n + 1$ $\quad x_j := \dfrac{j}{60 \cdot n}$ $\quad (ft - it) \cdot n = 1 \cdot 10^3$ $\quad n = 10$

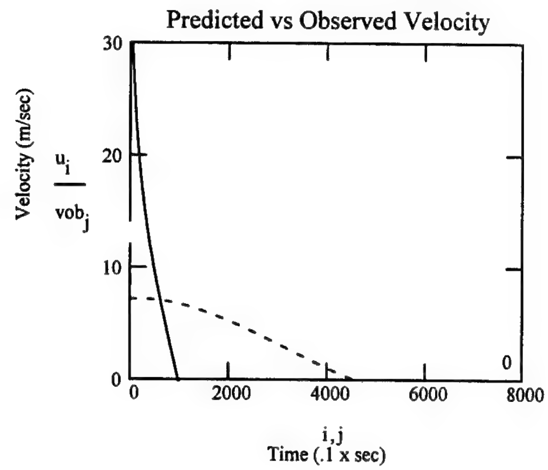
$zob_j := .2675 \cdot (x_j)^4 - 5.643 \cdot (x_j)^3 + 5.1671 \cdot (x_j)^2 + 428.75 \cdot x_j - 12.172$ $\quad \max(zob) = 1.96 \cdot 10^3$

$vob_j := \left[1.07 \cdot (x_j)^3 - 16.929 \cdot (x_j)^2 + 10.3342 \cdot x_j + 428.75 \right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $\quad u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot \sec \cdot m^{-1}$ $\quad \alpha = 0.364$

$H_i := Hu_i \cdot v4u$



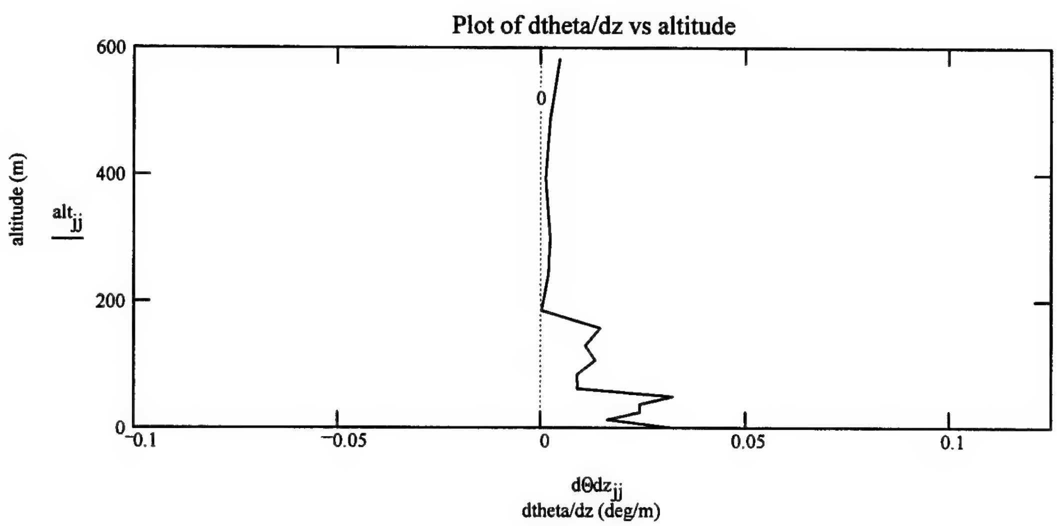


Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1 .. metf - 1$

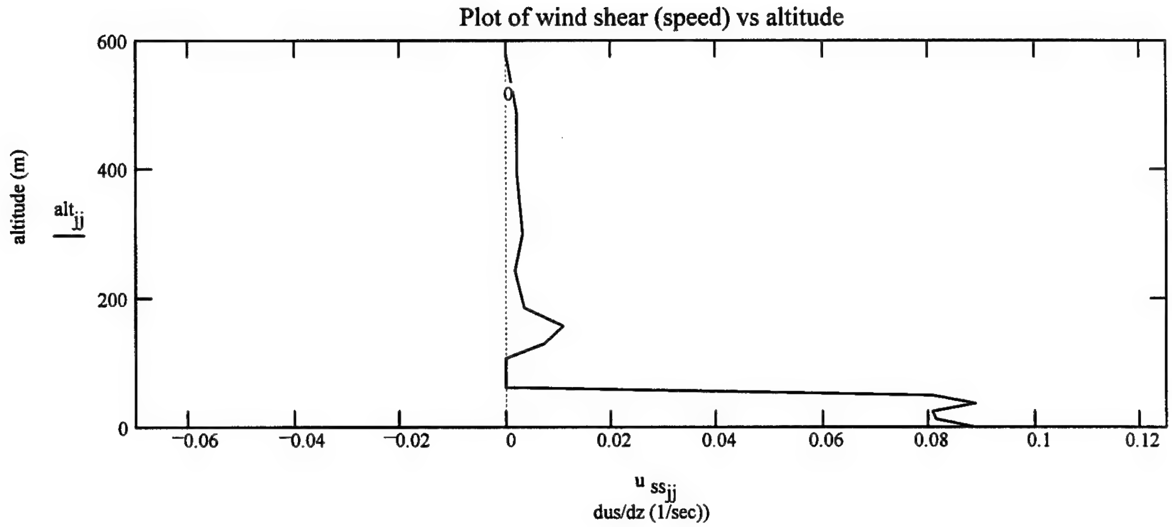
Determination of dtheta/dz:

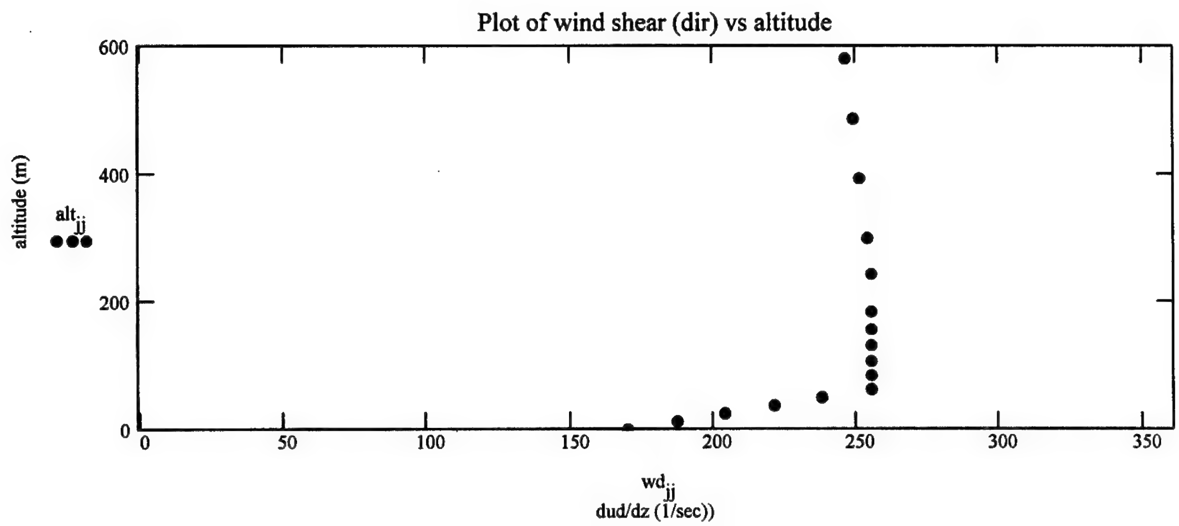
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}} \right) \cdot m \cdot K^{-1}$$



Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}} \cdot sec \qquad u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}} \cdot m \cdot deg^{-1}$$



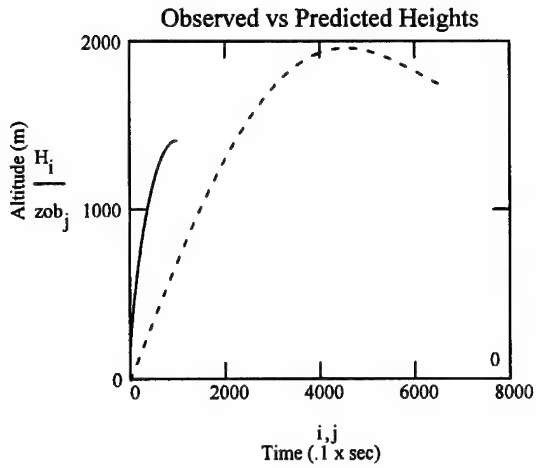


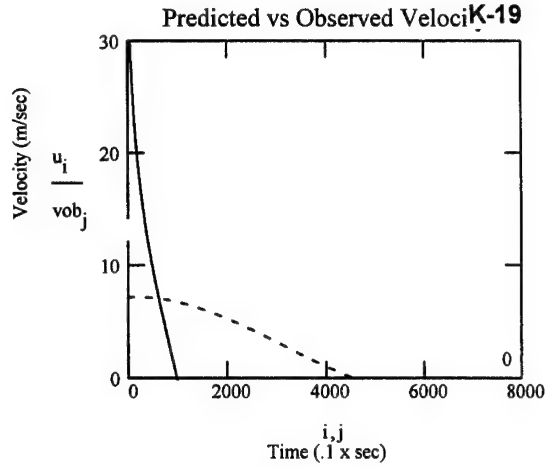
$ptj_{jj} := pt_{jj}$ $\quad$ $wsj_{jj} := ws_{jj}$ $\quad$ $wdj_{jj} := wd_{jj}$

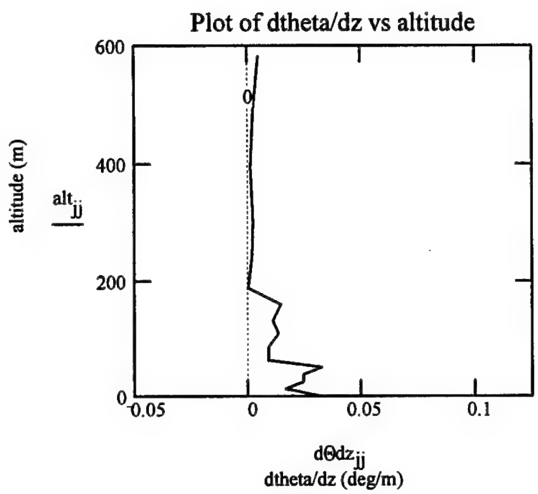
$mean\_pt := mean(ptj)$ $\quad$ $var\_pt := var(ptj)$

$mean\_ws := mean(wsj)$ $\quad$ $var\_ws := var(wsj)$

$mean\_wd := mean(wdj)$ $\quad$ $var\_wd := var(wdj)$







$mean\_pt = 301.95 \cdot K$

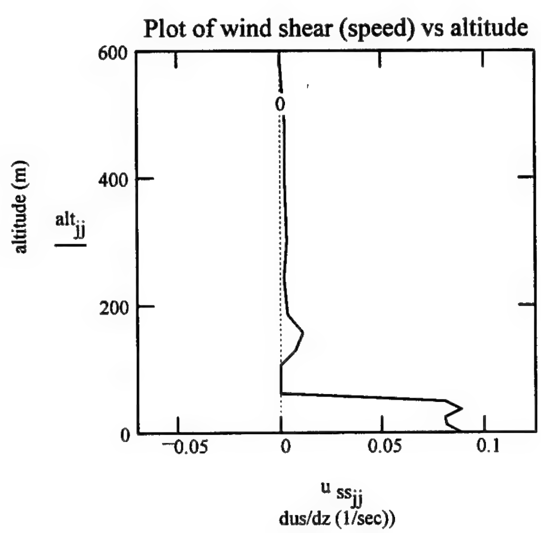
$var\_pt = 1.275 \cdot K^2$

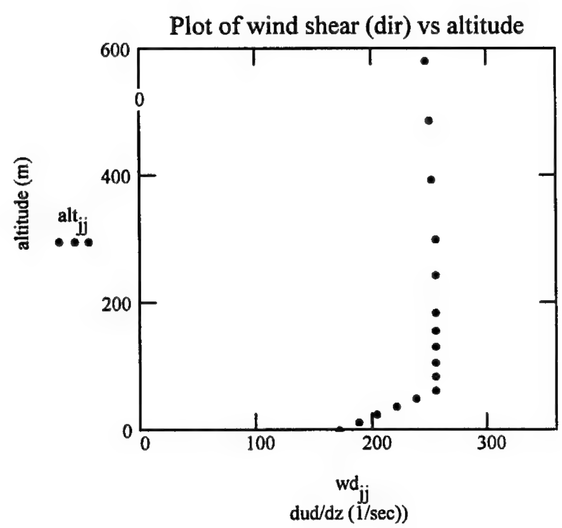
$mean\_ws = 6.113 \cdot m \cdot sec^{-1}$

$var\_ws = 3.745 \cdot m^2 \cdot sec^{-2}$

$mean\_wd = 237.813$

$var\_wd = 706.402$





Meteorological Data K-21: ORIGIN ≡ 1 **K-21**

d21 :=

1	16	0.0	0.0	20	1.5	3.0	17.3	17.3	15.0	1021.8	86.0
2	65	49.0	14.9	32	2.1	4.0	18.7	19.0	15.9	1020.0	84.0
3	114	98.0	29.9	45	2.6	5.0	20.0	20.6	16.9	1018.3	81.8
4	163	147.0	44.8	57	3.1	6.0	21.4	22.3	17.8	1016.5	79.7
5	212	196.0	59.7	69	3.6	7.0	22.8	24.0	18.7	1014.8	78.0
6	317	300.7	91.7	70	3.6	7.0	22.5	23.9	18.1	1011.1	76.2
7	421	405.5	123.6	71	3.6	7.0	22.2	23.8	17.5	1007.4	74.7
8	526	510.2	155.5	72	3.6	7.0	21.9	23.7	16.9	1003.7	73.3
9	631	615.0	187.5	73	3.6	7.0	21.6	23.7	16.3	1000.0	72.0
10	816	799.5	243.7	74	3.7	7.1	21.1	23.7	15.9	993.6	72.5
11	1000	984.0	299.9	74	3.7	7.2	20.6	23.7	15.6	987.2	73.0
12	1333	1317.3	401.5	77	3.5	6.7	19.5	23.6	15.2	975.6	76.3
13	1667	1650.7	503.1	79	3.2	6.3	18.5	23.5	14.9	964.2	79.6
14	2000	1984.0	604.7	82	3.0	5.8	17.4	23.4	14.5	952.9	83.0
15	2083	2067.0	630.0	84	3.1	6.0	17.1	23.4	14.5	950.0	85.0
16	2261	2245.0	684.3	87	2.6	5.0	16.5	23.3	14.5	944.1	88.0
17	2832	2816.0	858.3	106	2.1	4.0	15.0	23.5	14.3	925.0	96.0
18	3000	2984.0	909.5	115	1.9	3.6	14.4	23.3	13.7	919.5	95.0
19	3219	3203.0	976.3	131	1.5	3.0	13.7	23.2	12.9	912.3	95.0
20	3589	3573.0	1089.1	149	1.0	2.0	12.9	23.5	12.0	900.0	95.0
21	3786	3770.0	1149.1	166	1.0	2.0	12.5	23.6	11.6	893.8	94.0
22	3857	3841.3	1170.8	186	1.0	2.0	12.3	23.6	11.5	891.5	94.5
23	3929	3912.7	1192.6	205	1.0	2.0	12.2	23.7	11.3	889.2	94.7
24	4000	3984.0	1214.3	225	1.0	1.9	12.0	23.7	11.2	886.9	95.0
25	4091	4074.5	1241.9	241	1.0	2.0	11.8	23.8	11.0	884.0	95.1
26	4271	4255.5	1297.1	274	1.0	2.0	11.4	23.9	10.7	878.2	95.4
27	4907	4891.0	1490.8	304	3.6	7.0	11.4	25.2	2.1	858.2	53.0
28	5161	5145.0	1568.2	303	5.1	10.0	11.7	26.1	-2.4	850.0	39.0
29	6000	5984.0	1823.9	298	8.4	16.3	11.0	27.7	-7.1	824.7	27.0
30	6819	6803.0	2073.6	290	10.3	20.0	9.7	28.9	-9.2	800.0	26.0
31	7195	7179.0	2188.2	287	10.8	21.0	9.4	29.7	-11.8	789.4	21.0
32	8000	7984.0	2433.5	284	11.9	23.2	7.5	30.9	1.7	766.3	67.0
33	8569	8553.0	2607	283	12.3	24.0	7.0	32.4	3.5	750.0	78.0
34	9500	9484.0	2890.7	285	12.9	25.0	5.9	34.1	1.8	725.0	75.3

$rd := rows(d21)$

$i := 1 .. rd$

$ML_i := d21_{i,1}$

$alt_i := d21_{i,4} \cdot m$

$wd_i := d21_{i,5}$

$ws_i := d21_{i,6} \cdot m \cdot sec^{-1}$

$t_i := \left(d21_{i,8} + 273\right) \cdot K$

$pt_i := \left(d21_{i,9} + 273\right) \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d21_{i,11} \cdot mb$

K-21 Plume Rise Model

Model parameters:

$\alpha := .431$ $g = 9.807 \cdot m \cdot sec^{-2}$ $R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 6$ $alt_{meti} = 91.7 \cdot m$

Temp: $t_{meti} = 295.5 \cdot K$ **Pres:** $p_{meti} = 1.011 \cdot 10^5 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\rho_{00} = 1.192 \cdot kg \cdot m^{-3}$ $\rho_1 := \rho_{00}$

Final density: $metf := 14$ $alt_{metf} = 604.7 \cdot m$

Temp: $t_{metf} = 290.4 \cdot K$ **Pres:** $p_{metf} = 9.529 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\rho_{0f} = 1.143 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right)$ $d\rho dz = -9.531 \cdot 10^{-5} \cdot kg \cdot m^{-4}$ $d\rho dz := -9.6 \cdot 10^{-5}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $G = 7.897 \cdot 10^{-4} \cdot kg^{-1} \cdot m^4 \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $v3u := sec^2 \cdot m^{-4}$ $v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0^{\,3} \cdot v1u$ $v_2 := r_0^{\,3} \cdot \rho_i \cdot u_0 \cdot v2u$ $v_3 := r_0^{\,3} \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u$ $v_4 := 0 \cdot m \cdot v4u$

$v^T = (4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0)$

Determination of critical values:

$$rhs(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $\quad n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := rkfixed(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{\langle 2 \rangle} \cdot \dfrac{1}{v1u}$ $\quad rad_i := (v1_i)^{\frac{1}{3}}$ $\quad Hu := z^{\langle 5 \rangle} \cdot \dfrac{1}{v4u} + r_0$ $\quad Hu_m := max(Hu)$

$max(rad) = 673 \cdot m$ **<== Max plume radius**

$Hu_m = 1338 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{\langle 3 \rangle} \cdot \dfrac{1}{v2u}$ $\quad v3 := z^{\langle 4 \rangle} \cdot \dfrac{1}{v3u}$

Observed Cloud Rise Data:

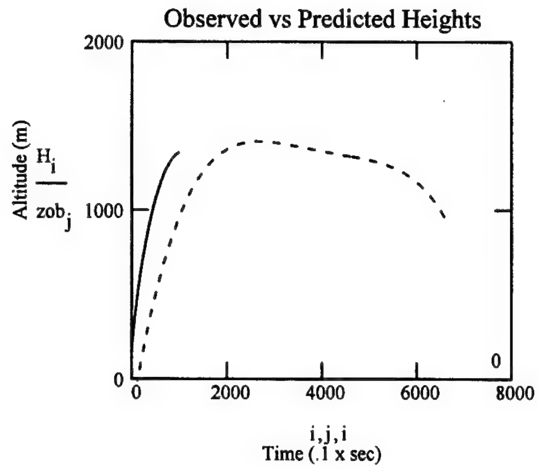
$t_{ob} := 11 \cdot 60$ $\quad j := 1 .. t_{ob} \cdot n + 1$ $\quad x_j := \dfrac{j}{60 \cdot n}$ $\quad (ft - it) \cdot n = 1 \cdot 10^3$ $\quad n = 10$

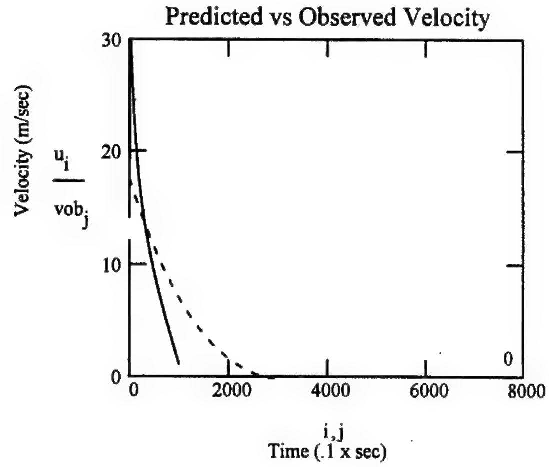
$zob_j := -0.9109 \cdot (x_j)^4 + 24.561 \cdot (x_j)^3 - 245.91 \cdot (x_j)^2 + 1051.9 \cdot x_j - 214.94$ $\quad max(zob) = 1.404 \cdot 10^3$

$vob_j := \left[-3.6436 \cdot (x_j)^3 + 73.683 \cdot (x_j)^2 - 491.82 \cdot x_j + 1051.9 \right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $\quad u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\quad \alpha = 0.431$

$H_i := Hu_i \cdot v4u$



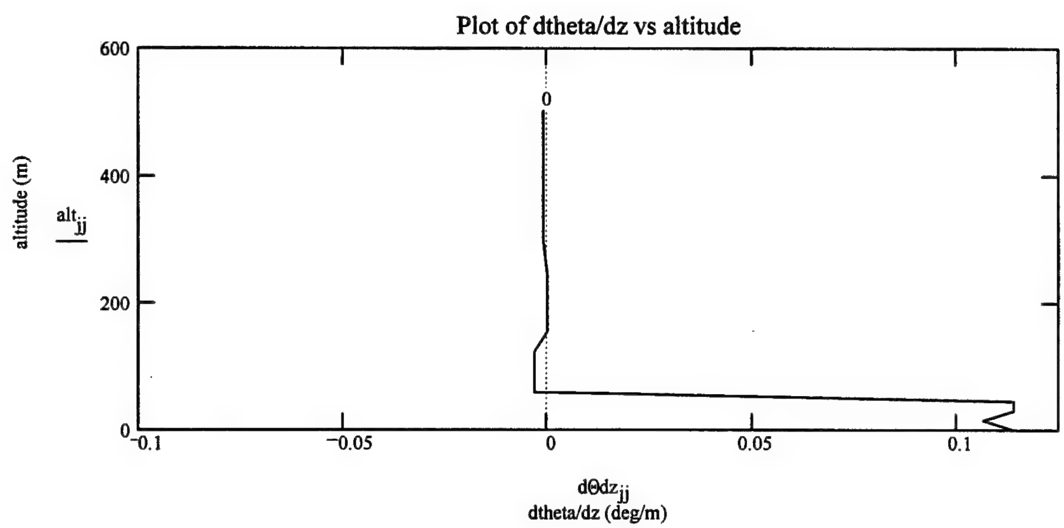


Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1 .. metf - 1$

Determination of dtheta/dz:

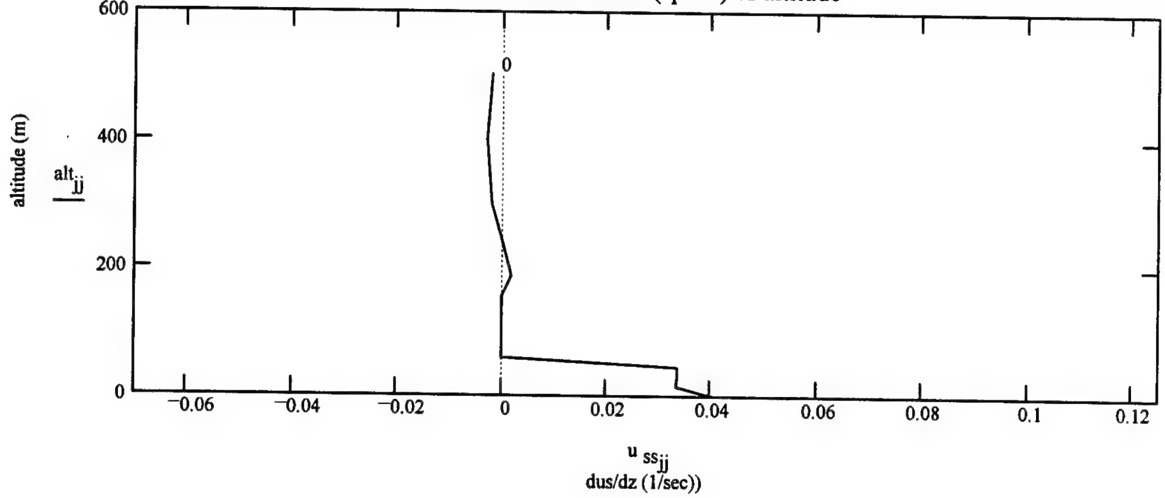
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}}\right)\cdot m\cdot K^{-1}$$



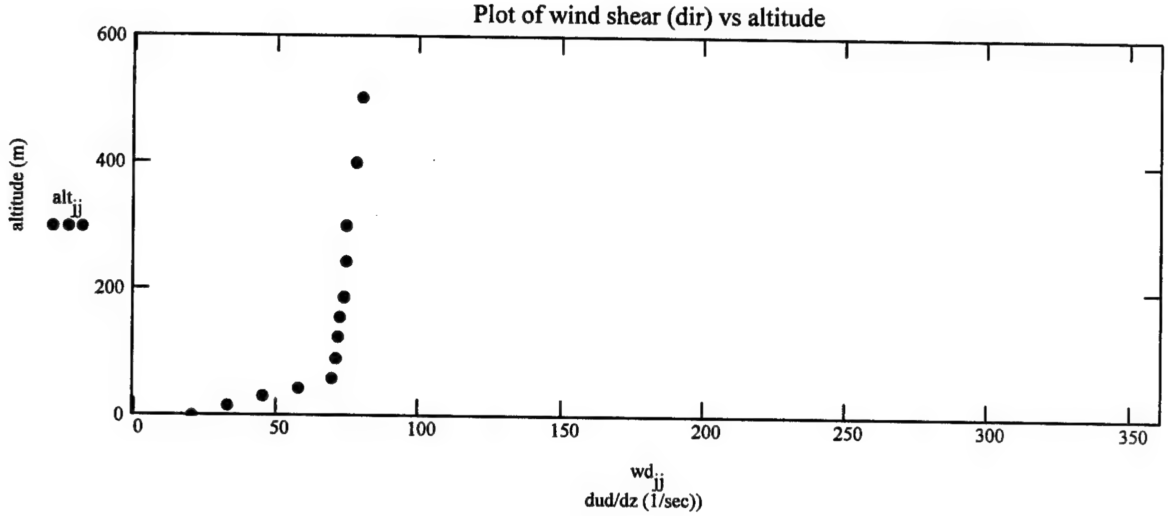
Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}}\cdot sec \qquad u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}}\cdot m\cdot deg^{-1}$$

Plot of wind shear (speed) vs altitude



Plot of wind shear (dir) vs altitude

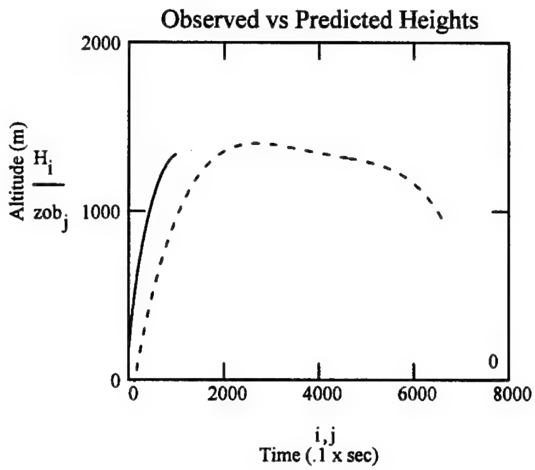


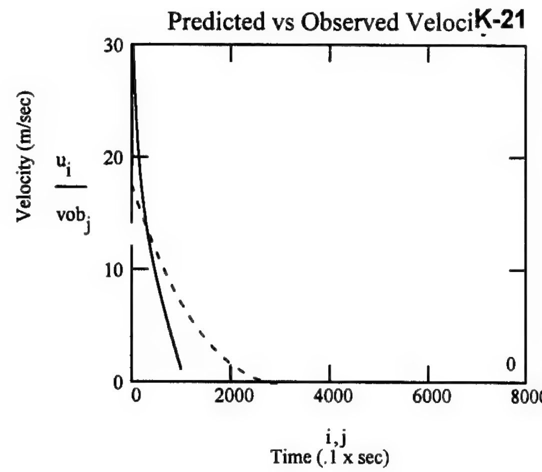
$ptj_{jj} := pt_{jj}$ $\quad wsj_{jj} := ws_{jj}$ $\quad wdj_{jj} := wd_{jj}$

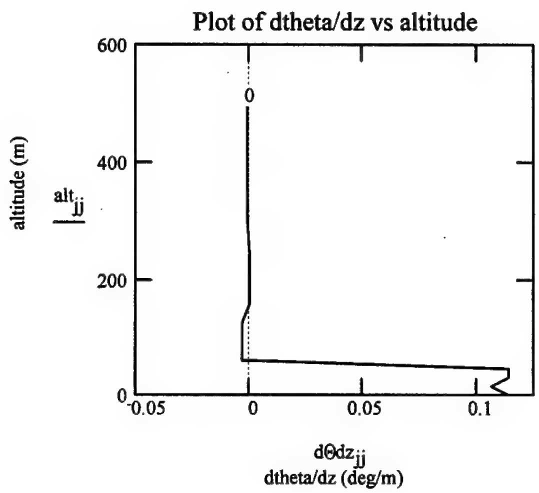
$mean\_pt := mean(ptj)$ $\quad var\_pt := var(ptj)$

$mean\_ws := mean(wsj)$ $\quad var\_ws := var(wsj)$

$mean\_wd := mean(wdj)$ $\quad var\_wd := var(wdj)$







mean_pt = 295.523•K

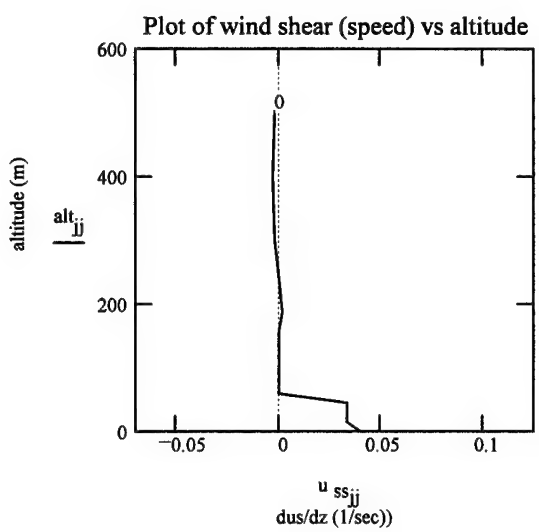
var_pt = 4.369•K^2

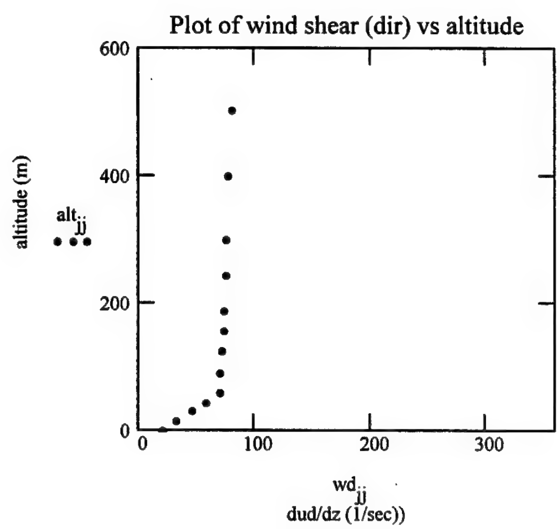
mean_ws = 3.185•m•sec^{-1}

var_ws = 0.451•m^2•sec^{-2}

mean_wd = 62.538

var_wd = 322.402





Meteorological Data K-23: ORIGIN ≡ 1 **K-23**

d23 :=

1	16	0.0	0.0	250	2.6	5.0	27.1	29.0	23.6	1016.4	81.0
2	63	46.8	14.3	251	2.7	5.2	26.8	28.8	23.4	1014.8	81.9
3	110	93.6	28.5	253	2.8	5.4	26.5	28.6	23.2	1013.2	82.5
4	156	140.4	42.8	254	2.9	5.6	26.1	28.3	23.1	1011.5	83.2
5	203	187.2	57.1	256	3.0	5.8	25.8	28.1	22.9	1009.9	83.8
6	250	234.0	71.3	257	3.1	6.0	25.5	27.9	22.7	1008.3	84.0
7	329	313.3	95.5	253	3.3	6.3	25.4	28.1	22.6	1005.5	84.5
8	409	392.7	119.7	250	3.4	6.7	25.4	28.2	22.6	1002.8	84.5
9	488	472.0	143.9	246	3.6	7.0	25.3	28.4	22.5	1000.0	85.0
10	659	642.7	195.9	242	3.8	7.3	25.4	29.0	22.3	994.1	83.2
11	829	813.3	247.9	237	3.9	7.7	25.4	29.5	22.1	988.3	81.9
12	1000	984.0	299.9	233	4.1	8.0	25.5	30.1	21.9	982.5	81.0
13	1142	1126.0	343.2	231	4.1	8.0	25.7	30.7	21.6	977.8	78.0
14	1414	1398.0	426.1	229	4.4	8.5	25.5	31.2	20.9	968.7	75.4
15	1686	1670.0	509.0	227	4.6	9.0	25.4	31.8	20.1	959.6	72.0
16	2000	1684.0	604.7	225	5.0	9.7	24.8	32.1	19.8	949.3	74.0
17	2606	2590.0	789.4	225	5.1	10.0	23.7	32.7	18.9	929.6	75.0
18	3000	2984.0	909.5	225	5.0	9.7	22.6	32.8	18.4	916.9	77.0
19	3528	3512.0	1070.5	223	4.6	9.0	21.4	33.1	17.9	900.0	81.0
20	4000	3984.0	1214.3	223	4.0	7.8	20.1	33.2	17.5	885.4	85.0
21	4717	4701.0	1432.9	226	3.1	6.0	18.2	33.3	16.5	863.3	90.0
22	5000	4984.0	1519.1	230	2.5	4.8	17.9	33.7	15.3	854.7	85.0
23	5147	5131.0	1563.9	233	2.1	4.0	17.7	33.9	14.7	850.0	83.0
24	5574	5557.5	1693.9	245	1.6	3.2	17.0	34.3	13.2	837.3	78.6
25	6000	5984.0	1823.9	256	1.2	2.4	16.3	34.7	11.7	824.9	74.0
26	6323	6307.0	1922.4	269	1.0	2.0	15.5	34.8	10.8	815.4	74.0
27	6497	6480.7	1975.3	289	1.0	2.0	15.3	35.1	9.6	810.2	69.1
28	6670	6654.3	2028.2	308	1.0	2.0	15.1	35.3	8.3	805.1	64.5
29	6844	6828.0	2081.2	328	1.0	1.9	14.9	35.5	7.1	800.0	59.0
30	6922	6909.0	2104.9	342	1.0	1.9	14.8	35.5	6.2	797.9	57.0
31	7000	6984.0	2128.7	357	1.0	1.9	14.7	35.6	5.3	795.9	54.0
32	7427	7411.0	2258.9	11	1.0	1.9	14.2	36.0	-0.3	783.8	37.0
33	8628	8612.0	2624.9	22	1.0	1.9	11.9	37.6	1.6	750.0	49.0
34	9500	9484.0	2890.7	29	1.0	1.9	10.6	39.0	0.8	727.0	51.5

$rd := rows(d23)$

$i := 1 .. rd$

$ML_i := d23_{i,1}$

$alt_i := d23_{i,4} \cdot m$

$wd_i := d23_{i,5}$

$ws_i := d23_{i,6} \cdot m \cdot sec^{-1}$

$t_i := \left(d23_{i,8} + 273\right) \cdot K$

$pt_i := \left(d23_{i,9} + 273\right) \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d23_{i,11} \cdot mb$

K-23 Plume Rise Model

Model parameters:

$\alpha := .252$ $\quad g = 9.807 \cdot m \cdot sec^{-2}$ $\quad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $\quad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\quad \rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 6$ $\quad alt_{meti} = 71.3 \cdot m$

Temp: $t_{meti} = 298.5 \cdot K$ **Pres:** $p_{meti} = 1.008 \cdot 10^5 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\quad \rho_{00} = 1.177 \cdot kg \cdot m^{-3}$ $\quad \rho_1 := \rho_{00}$

Final density: $metf := 16$ $\quad alt_{metf} = 604.7 \cdot m$

Temp: $t_{metf} = 297.8 \cdot K$ **Pres:** $p_{metf} = 9.493 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\quad \rho_{0f} = 1.111 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}}\right)$ $\quad d\rho dz = -1.242 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $\quad G = 1.035 \cdot 10^{-3} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $\quad v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $\quad v3u := sec^2 \cdot m^{-4}$ $\quad v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0^{\,3} \cdot v1u$ $\quad v_2 := r_0^{\,3} \cdot \rho_i \cdot u_0 \cdot v2u$ $\quad v_3 := r_0^{\,3} \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1}\right) \cdot v3u$ $\quad v_4 := 0 \cdot m \cdot v4u$

$v^T = (\, 4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0 \,)$

Determination of critical values:

$$\mathrm{rhs}(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow - G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $\quad n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := \mathrm{rkfixed}(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{\langle 2 \rangle} \cdot \dfrac{1}{v1u}$ $\quad rad_i := (v1_i)^{\frac{1}{3}}$ $\quad Hu := z^{\langle 5 \rangle} \cdot \dfrac{1}{v4u} + r_0$ $\quad Hu_m := \max(Hu)$

$\max(rad) = 551 \cdot m$ **<== Max plume radius**

$Hu_m = 1682 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{\langle 3 \rangle} \cdot \dfrac{1}{v2u}$ $\quad v3 := z^{\langle 4 \rangle} \cdot \dfrac{1}{v3u}$

Observed Cloud Rise Data:

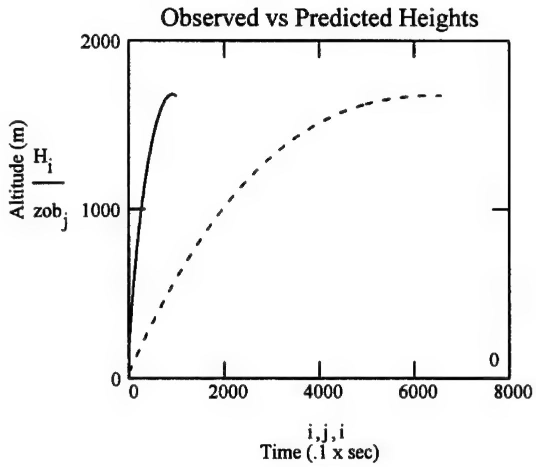
$t_{ob} := 11 \cdot 60$ $\quad j := 1 .. t_{ob} \cdot n + 1$ $\quad x_j := \dfrac{j}{60 \cdot n}$ $\quad (ft - it) \cdot n = 1 \cdot 10^3$ $\quad n = 10$

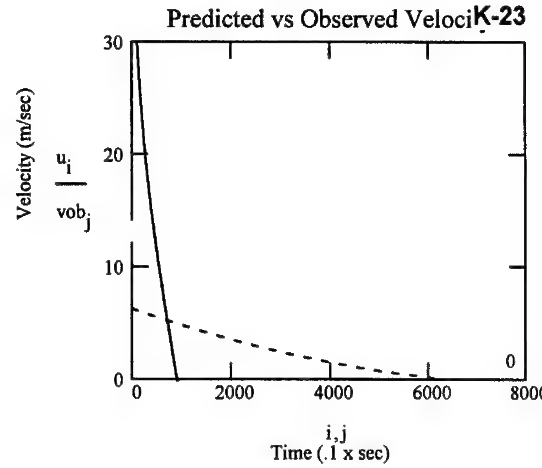
$zob_j := 0.6149 \cdot (x_j)^3 - 27.571 \cdot (x_j)^2 + 377.6 \cdot x_j + 38.08$ $\quad \max(zob) = 1.675 \cdot 10^3$

$vob_j := \left[1.8447 \cdot (x_j)^2 - 55.142 \cdot x_j + 377.6 \right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $\quad u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\quad \alpha = 0.252$

$H_i := Hu_i \cdot v4u$





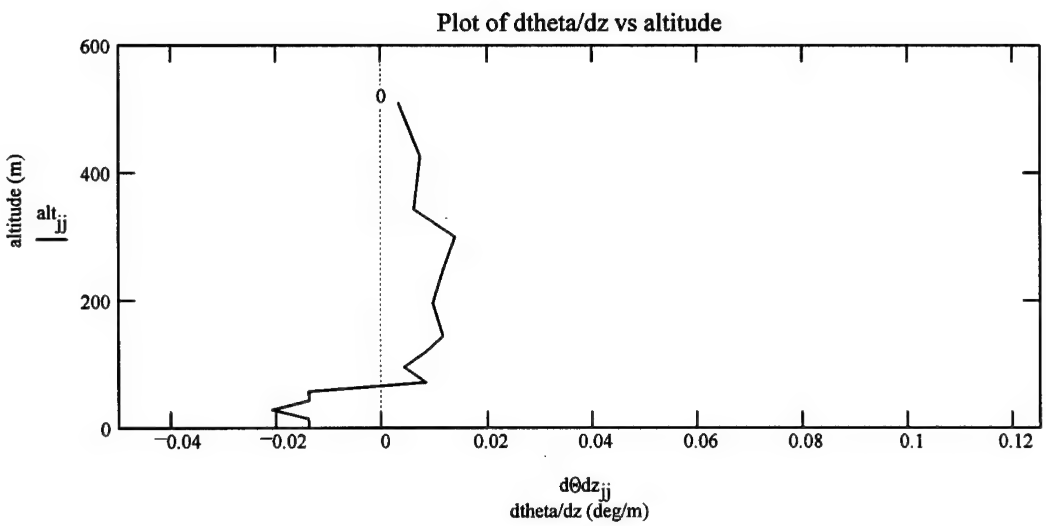
Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1 .. metf - 1$

Determination of dtheta/dz:

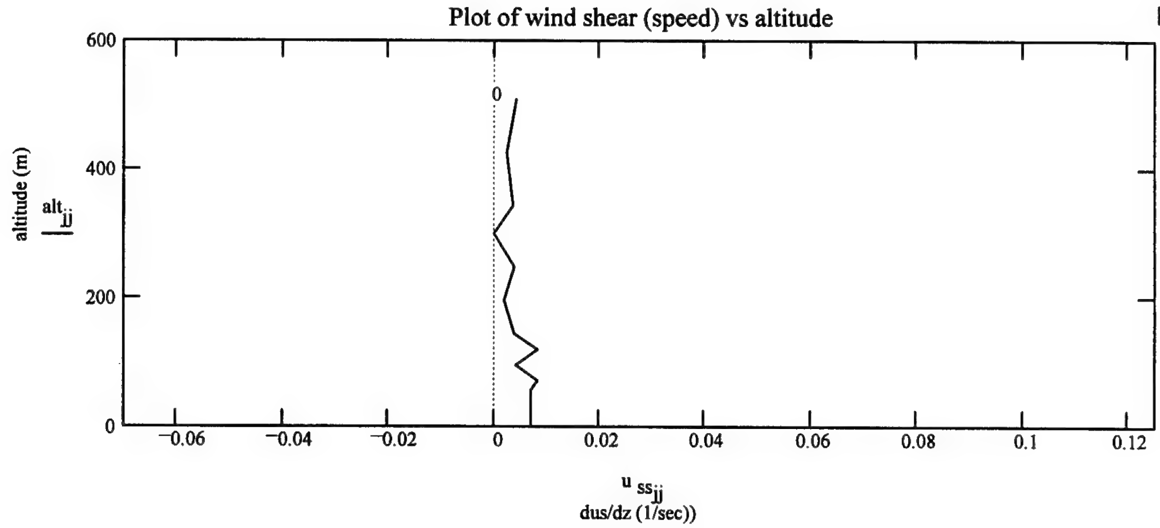
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}} \right) \cdot m \cdot K^{-1}$$

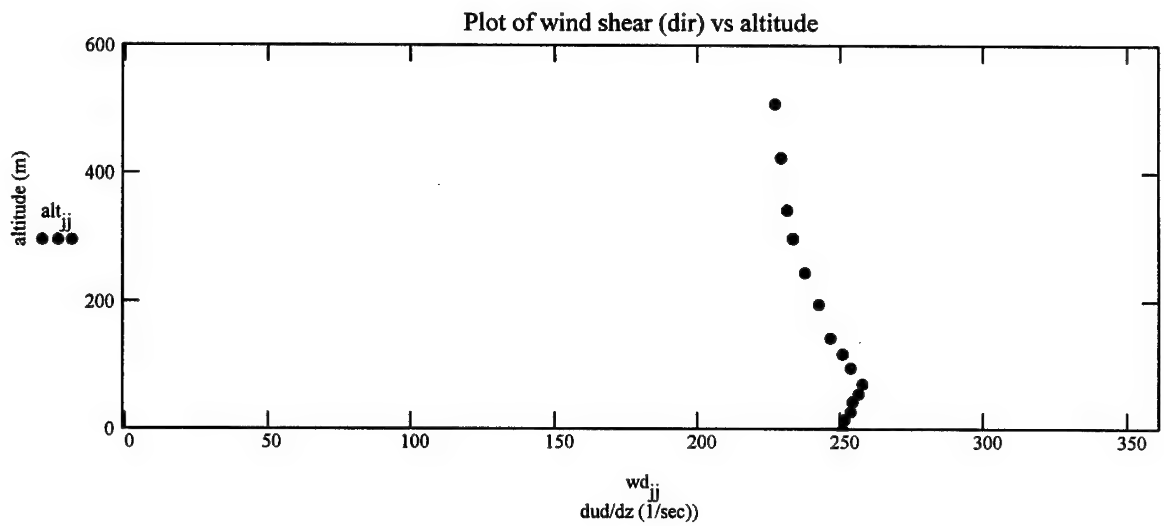
$pt_2 = 301.8 \cdot K$ $\quad pt_1 = 302 \cdot K$



Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}} \cdot sec \qquad u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}} \cdot m \cdot deg^{-1}$$



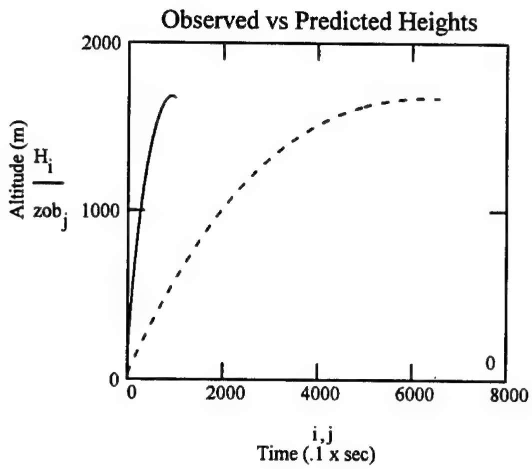


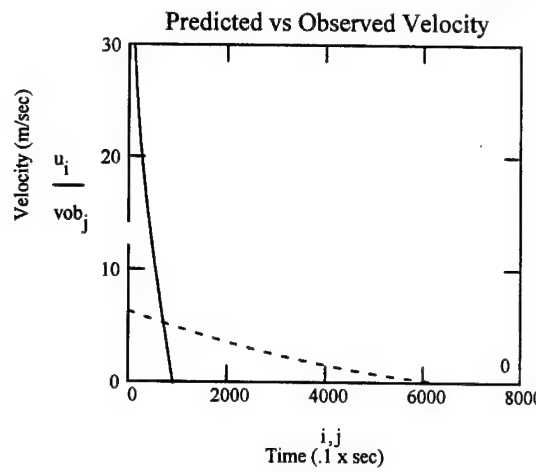
$ptj_{jj} := pt_{jj}$ $\quad wsj_{jj} := ws_{jj}$ $\quad wdj_{jj} := wd_{jj}$

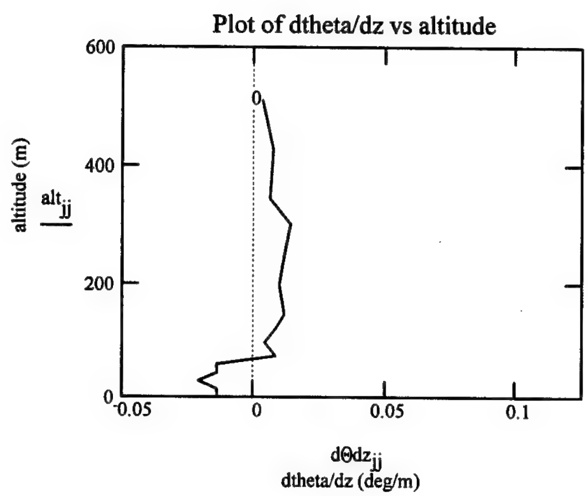
$mean\_pt := mean(ptj)$ $\quad var\_pt := var(ptj)$

$mean\_ws := mean(wsj)$ $\quad var\_ws := var(wsj)$

$mean\_wd := mean(wdj)$ $\quad var\_wd := var(wdj)$







mean_pt = 302.18•K

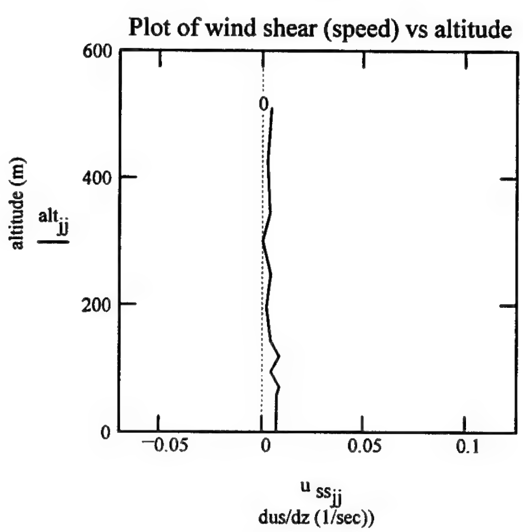
var_pt = 1.404•K^2

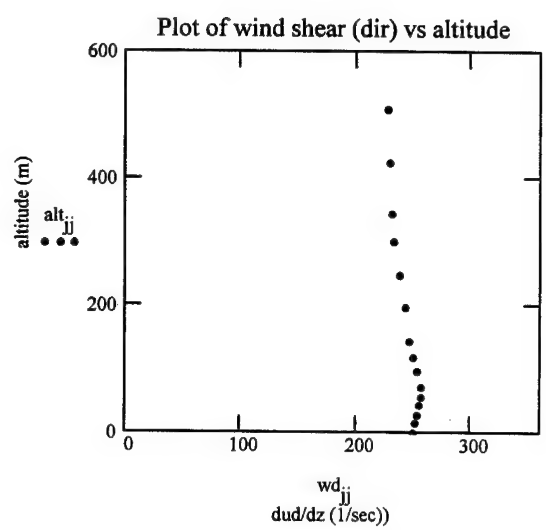
mean_ws = 3.487•m•sec^{-1}

var_ws = 0.384•m^2•sec^{-2}

mean_wd = 244.6

var_wd = 103.44





Meteorological Data K-15: ORIGIN ≡ 1

d15 :=

1	368	0.0	0.0	0	4.1	8.0	17.8	19.0	11.2	1002.9	65.0
2	422	54.0	16.5	13	5.1	10.0	16.9	18.3	11.0	1001.0	68.0
4	493	125.0	38.1	7	4.7	9.2	15.8	17.4	10.8	998.5	72.0
6	572	204.0	62.2	358	5.1	10.0	15.6	17.4	11.2	995.6	74.8
8	668	300.0	91.4	341	6.7	13.0	15.3	17.5	11.6	992.2	78.3
9	716	348.0	106.1	346	8.3	16.1	15.2	17.5	11.8	990.5	80.0
10	821	453.0	138.1	346	10.8	20.9	17.9	20.8	13.4	986.8	75.0
12	1024	656.0	199.9	353	9.9	19.2	18.2	21.6	12.8	979.7	71.1
13	1188	820.0	249.9	0	8.4	16.3	18.5	22.4	12.4	974.0	67.9
14	1393	1025.0	312.4	17	8.5	16.6	18.8	23.2	11.8	967.0	63.9
15	1516	1148.0	349.9	25	10.2	19.8	18.7	23.5	11.7	962.8	63.6
16	1680	1312.0	399.9	17	7.3	14.2	18.6	23.9	11.5	957.2	63.1
18	1844	1476.0	449.9	41	8.4	16.3	18.6	24.4	11.3	951.7	62.6
20	2008	1640.0	499.9	11	5.7	11.1	18.5	24.8	11.1	946.1	62.2
21	2518	2150.0	655.3	26	8.9	17.3	18.2	26.0	10.5	929.2	60.7
23	3629	3261.0	994.0	360	8.0	15.5	16.7	27.6	7.1	893.1	53.2
25	4768	4400.0	1341.1	356	7.7	14.9	14.0	28.2	5.4	857.4	55.9
27	5884	5516.0	1681.3	346	7.4	14.3	13.6	31.1	1.8	823.5	44.6
28	6470	6102.0	1859.9	332	7.5	14.6	12.1	31.4	2.6	806.2	52.0
29	6927	6559.0	1999.2	319	7.4	14.4	11.7	32.1	-2.5	793.0	37.0
30	7889	7521.0	2292.4	307	6.4	12.5	9.9	33.2	-4.6	765.6	35.6
32	8921	8553.0	2607.0	290	5.3	10.3	8.6	35.1	-5.6	737.1	35.9
34	10041	9673.0	2498.3	280	4.8	9.4	5.8	35.7	-5.7	707.2	43.4

$rd := \mathrm{rows}(d15)$

$i := 1 .. rd$

$ML_i := d15_{i,1}$

$alt_i := d15_{i,4} \cdot m$

$wd_i := d15_{i,5}$

$ws_i := d15_{i,6} \cdot m \cdot sec^{-1}$

$t_i := \left(d15_{i,8} + 273\right) \cdot K$

$pt_i := \left(d15_{i,9} + 273\right) \cdot K$

$mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$p_i := d15_{i,11} \cdot mb$

K-15 Plume Rise Model

Model parameters:

$\alpha := .959$ $g = 9.807 \cdot m \cdot sec^{-2}$ $R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 4$

Temp: $t_{meti} = 288.6 \cdot K$ **Pres:** $p_{meti} = 9.956 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\rho_{00} = 1.202 \cdot kg \cdot m^{-3}$ $\rho_1 := \rho_{00}$

Final density: $metf := 21$

Temp: $t_{metf} = 282.9 \cdot K$ **Pres:** $p_{metf} = 7.656 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\rho_{0f} = 0.943 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right)$ $d\rho dz = -1.162 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $G = 9.477 \cdot 10^{-4} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $v3u := sec^2 \cdot m^{-4}$ $v4u := m^{-1}$

Initial Conditions:

$v_1 := r_0^{\,3} \cdot v1u$ $v_2 := r_0^{\,3} \cdot \rho_i \cdot u_0 \cdot v2u$ $v_3 := r_0^{\,3} \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u$ $v_4 := 0 \cdot m \cdot v4u$

$v^T = (\, 4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0 \,)$

Determination of critical values:

$$rhs(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := rkfixed(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{<2>} \cdot \dfrac{1}{v1u}$ $rad_i := (v1_i)^{\frac{1}{3}}$ $Hu := z^{<5>} \cdot \dfrac{1}{v4u} + r_0$ $Hu_m := \max(Hu)$

$\max(rad) = 788 \cdot m$ **<== Max plume radius**

$Hu_m = 816 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{<3>} \cdot \dfrac{1}{v2u}$ $v3 := z^{<4>} \cdot \dfrac{1}{v3u}$

Observed Cloud Rise Data:

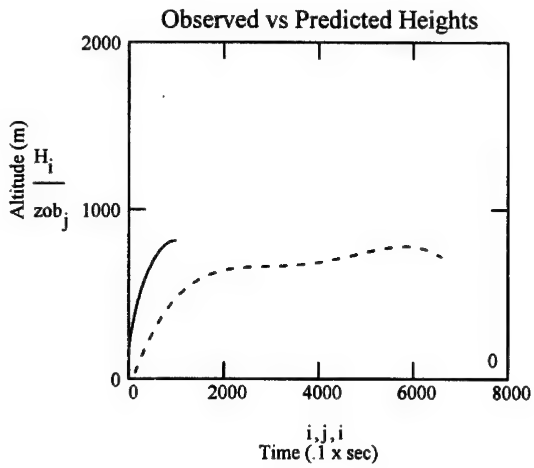
$t_{ob} := 11 \cdot 60$ $j := 1 .. t_{ob} \cdot n + 1$ $x_j := \dfrac{j}{60 \cdot n}$ $(ft - it) \cdot n = 1 \cdot 10^3$ $n = 10$

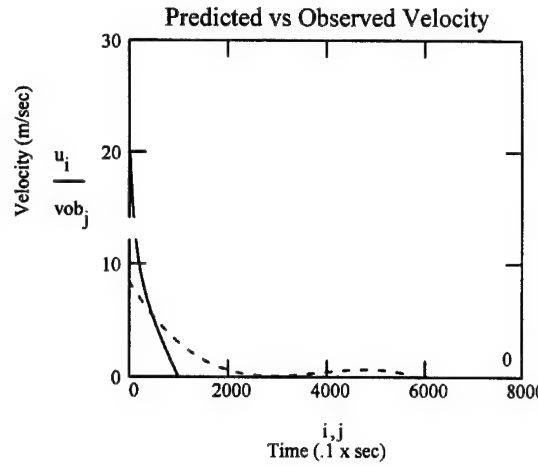
$zob_j := -0.5498 \cdot (x_j)^4 + 14.152 \cdot (x_j)^3 - 128.34 \cdot (x_j)^2 + 502.52 \cdot x_j - 66.388$ $\max(zob) = 781.354$

$vob_j := \left[-2.1992 \cdot (x_j)^3 + 42.456 \cdot (x_j)^2 - 256.68 \cdot x_j + 502.52\right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\alpha = 0.959$

$H_i := Hu_i \cdot v4u$



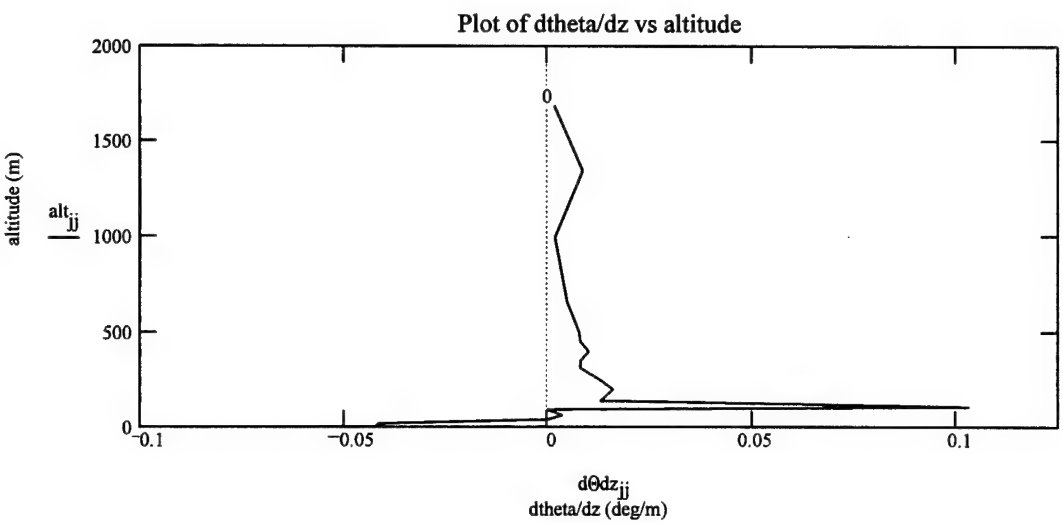


Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1 .. 18$

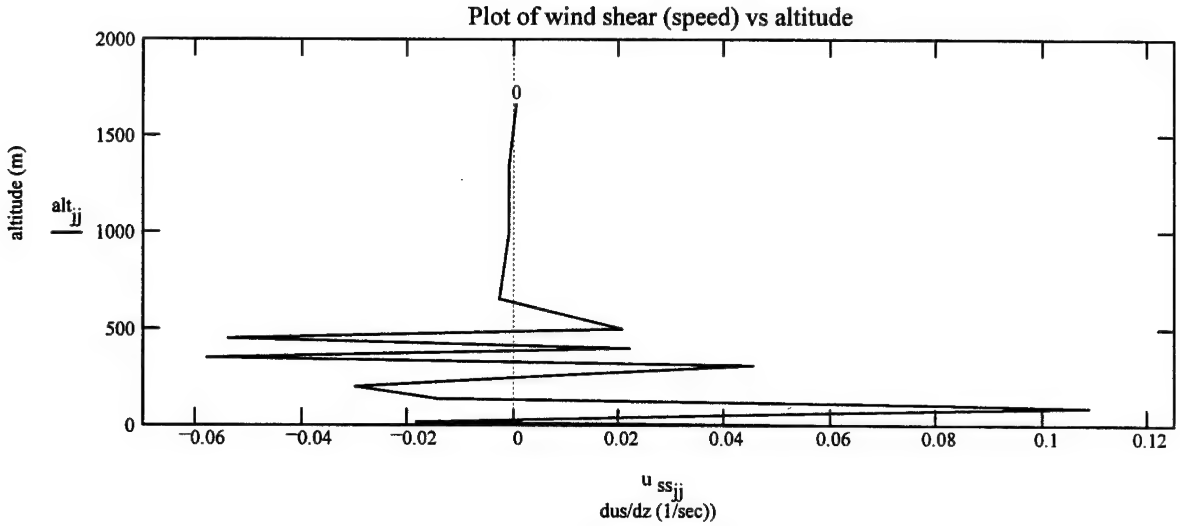
Determination of dtheta/dz:

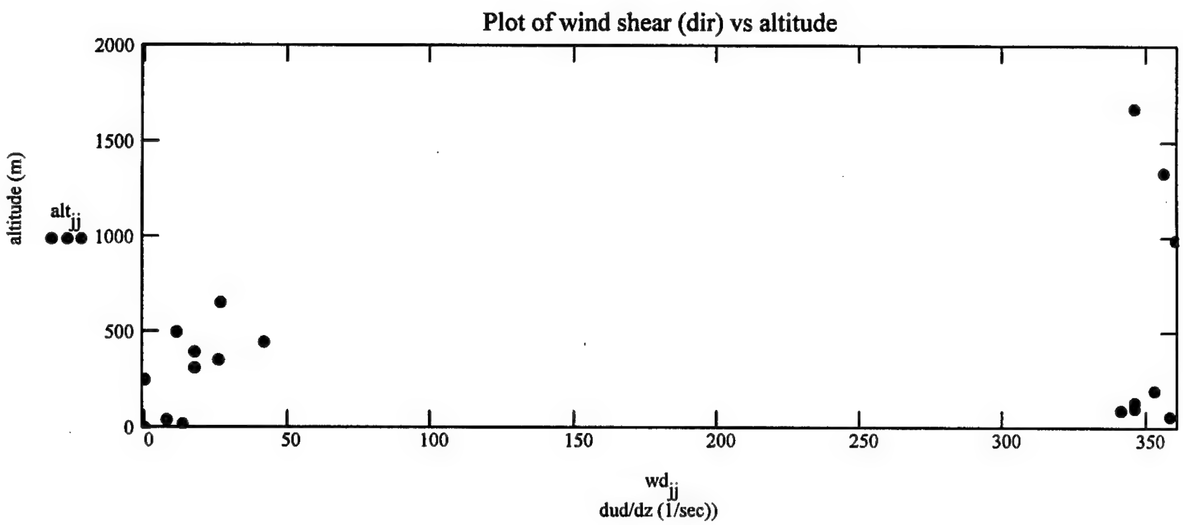
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}}\right) \cdot m \cdot K^{-1}$$



Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}} \cdot sec \qquad u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}} \cdot m \cdot deg^{-1}$$



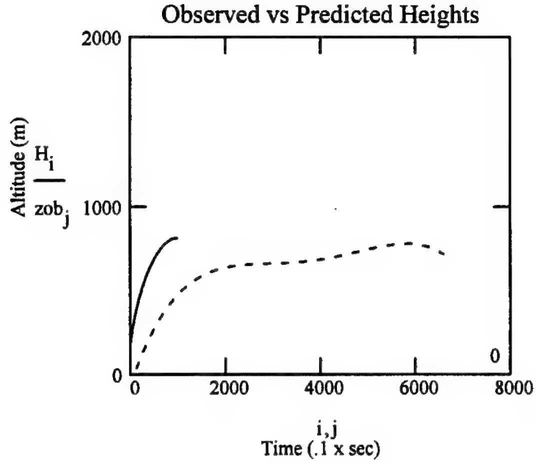


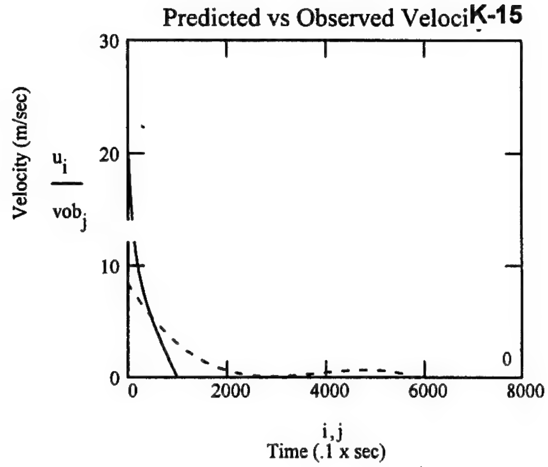
$ptj_{jj} := pt_{jj}$ $\quad$ $wsj_{jj} := ws_{jj}$ $\quad$ $wdj_{jj} := wd_{jj}$

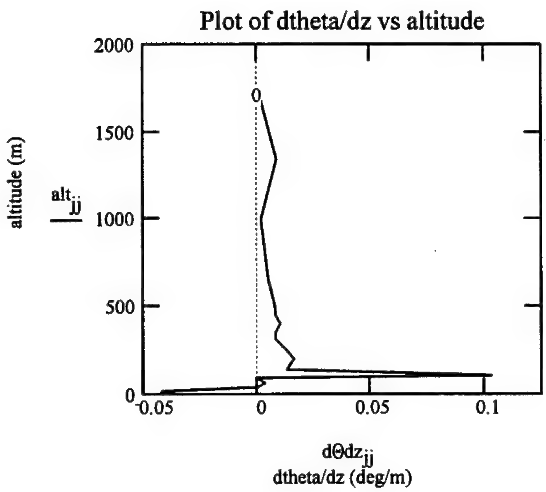
mean_pt := mean(ptj) $\quad$ var_pt := var(ptj)

mean_ws := mean(wsj) $\quad$ var_ws := var(wsj)

mean_wd := mean(wdj) $\quad$ var_wd := var(wdj)







$$mean\_pt = 295.478 \cdot K$$

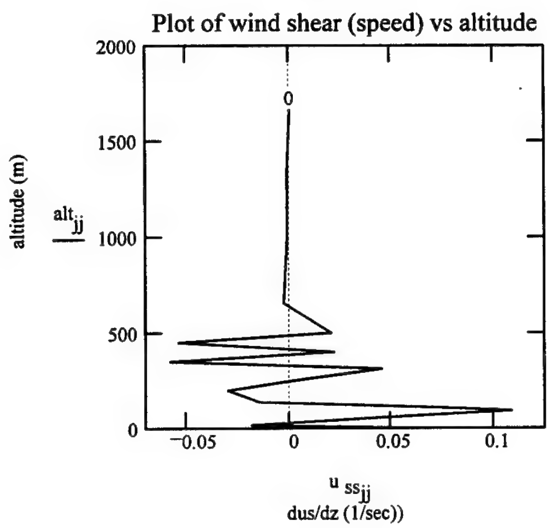
$$var\_pt = 16.26 \cdot K^2$$

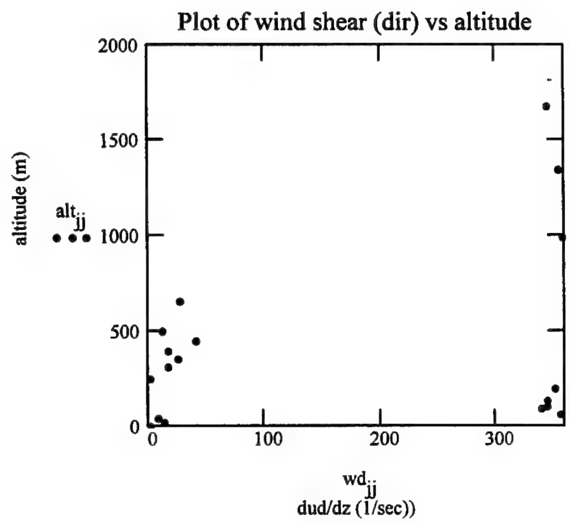
$$mean\_ws = 7.511 \cdot m \cdot sec^{-1}$$

$$var\_ws = 3.572 \cdot m^2 \cdot sec^{-2}$$

$$mean\_wd = 164.611$$

$$var\_wd = 2.782 \cdot 10^4$$





Meteorological Data K-22:

ORIGIN ≡ 1

d22 :=

1	329.0	-39.0	-11.9	355.0	3.1	6.0	19.4	19.4	10.7	1001.8	57.0
2	383.0	15.0	4.6	0.0	3.1	6.0	18.7	18.7	9.9	999.9	56.0
3	431.0	63.0	19.2	355.0	3.1	6.0	18.1	18.1	9.2	998.2	56.4
4	513.0	145.0	44.2	354.3	3.1	6.0	17.0	17.0	8.0	995.3	56.0
5	620.0	252.0	76.8	359.0	2.6	5.0	16.7	16.7	10.3	991.5	66.0
6	829.0	461.0	140.5	345.0	3.4	6.6	15.0	15.1	9.4	984.1	69.0
7	984.0	616.0	187.8	343.0	4.8	9.3	15.2	15.3	9.8	978.6	70.0
8	1149.0	781.0	238.0	320.0	6.0	11.7	17.9	18.0	9.4	972.9	58.1
9	1262.0	894.0	272.5	295.9	4.9	9.6	19.8	20.0	9.1	969.0	50.0
10	1313.0	945.0	288.0	285.0	4.5	8.7	21.3	21.5	9.9	967.3	48.3
11	1384.0	1016.0	309.7	289.3	5.0	9.7	23.4	23.6	11.1	964.8	46.0
12	1477.0	1109.0	338.0	295.0	5.6	10.9	23.9	24.2	10.3	961.7	42.5
13	1553.0	1185.0	361.2	323.4	5.9	11.4	24.3	24.6	9.7	959.2	39.6
14	2059.0	1691.0	515.4	328.7	6.7	13.1	26.5	27.0	6.5	942.5	28.0
15	2700.0	2332.0	710.8	333.4	6.9	13.4	25.8	26.4	4.5	921.8	25.3
16	3884.0	3516.0	1071.7	340.1	5.5	10.7	23.9	24.8	0.7	884.6	21.6
17	5072.0	4704.0	1433.8	1.9	3.1	6.1	20.9	21.9	-1.2	848.5	22.7
18	6290.0	5922.0	1805.0	34.3	1.7	3.3	18.1	19.2	-1.9	812.7	25.5
19	7538.0	7170.0	2185.4	287.0	0.4	0.8	17.1	18.4	-8.3	777.3	16.8
20	8754.0	8386.0	2556.1	251.2	2.2	4.3	15.2	16.5	-10.0	744.2	16.6
21	10063.0	9695.0	2955.0	245.4	2.8	5.5	13.7	15.1	-11.0	709.9	16.9

$rd := \mathrm{rows}(d22)$ $\quad i := 1 .. rd$

$ML_i := d22_{i,1}$ $\quad ws_i := d22_{i,6} \cdot m \cdot sec^{-1}$ $\quad mb := 100 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$alt_i := d22_{i,4} \cdot m$ $\quad t_i := \left(d22_{i,8} + 273\right) \cdot K$ $\quad p_i := d22_{i,11} \cdot mb$

$wd_i := d22_{i,5}$ $\quad pt_i := \left(d22_{i,9} + 273\right) \cdot K$

K-22 Plume Rise Model

Model parameters:

$\alpha := 1.118$ $\quad g = 9.807 \cdot m \cdot sec^{-2}$ $\quad R := 287 \cdot m^2 \cdot sec^{-2} \cdot K^{-1}$ $\quad u_0 := 0 \cdot m \cdot sec^{-1}$

Mass of initial plume: $ms := 3.899 \cdot 10^4 \cdot kg$ **Radius of initial plume:** $r_0 := 170 \cdot m$

Density of initial plume: $\rho_i := \dfrac{3 \cdot ms}{4 \cdot \pi \cdot (r_0)^3}$ $\quad \rho_i = 1.895 \cdot 10^{-3} \cdot kg \cdot m^{-3}$

Ambient Air Density Calculations:

Initial density: $meti := 5$ $\quad alt_{meti} = 76.8 \cdot m$

Temp: $t_{meti} = 289.7 \cdot K$ **Pres:** $p_{meti} = 9.915 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{00} := \dfrac{p_{meti}}{R \cdot t_{meti}}$ $\quad \rho_{00} = 1.193 \cdot kg \cdot m^{-3}$ $\quad \rho_1 := \rho_{00}$

Final density: $metf := 14$ $\quad alt_{metf} = 515.4 \cdot m$

Temp: $t_{metf} = 299.5 \cdot K$ **Pres:** $p_{metf} = 9.425 \cdot 10^4 \cdot kg \cdot m^{-1} \cdot sec^{-2}$

$\rho_{0f} := \dfrac{p_{metf}}{R \cdot t_{metf}}$ $\quad \rho_{0f} = 1.096 \cdot kg \cdot m^{-3}$

$d\rho dz := \left(\dfrac{\rho_{0f} - \rho_{00}}{alt_{metf} - alt_{meti}} \right)$ $\quad d\rho dz = -2.189 \cdot 10^{-4} \cdot kg \cdot m^{-4}$

$G := -g \cdot \dfrac{d\rho dz}{\rho_1}$ $\quad G = 1.8 \cdot 10^{-3} \cdot sec^{-2}$

Unit Conversions:

$v1u := m^{-3}$ $\quad v2u := sec \cdot kg^{-1} \cdot m^{-1}$ $\quad v3u := sec^2 \cdot m^{-4}$ $\quad v4u := m^{-1}$

Initial Conditions:

$v_1 := {r_0}^3 \cdot v1u$ $\quad v_2 := {r_0}^3 \cdot \rho_i \cdot u_0 \cdot v2u$ $\quad v_3 := {r_0}^3 \cdot g \cdot \left(\dfrac{\rho_{00} - \rho_i}{\rho_1} \right) \cdot v3u$ $\quad v_4 := 0 \cdot m \cdot v4u$

$v^T = (4.913 \cdot 10^6 \quad 0 \quad 4.81 \cdot 10^7 \quad 0)$

Determination of critical values:

$$\text{rhs}(t,v) := \left| \begin{array}{l} \rho_0 \leftarrow \rho_{00} + d\rho dz \cdot v_4 \\ \rho \leftarrow \rho_0 - v_3 \cdot \dfrac{\rho_1}{g \cdot v_1} \\ u \leftarrow \dfrac{v_2}{\rho \cdot v_1} \\ r \leftarrow (v_1)^{\frac{1}{3}} \\ d_1 \leftarrow 3 \cdot r^2 \cdot \alpha \cdot u \\ d_2 \leftarrow \rho_1 \cdot v_3 \\ d_3 \leftarrow -G \cdot u \cdot v_1 \\ d_4 \leftarrow u \\ d \end{array} \right.$$

Note:
v1 = cloud radius (meters)
v4 = cloud height (meters)

Initial time value: $it := 0$ $\quad n := 10$

Final time value: $ft := 100$

# of time steps: $nt := (ft - it) \cdot n$

$z := \text{rkfixed}(v, it, ft, nt, rhs)$

$i := 1 .. nt + 1$

$v1 := z^{\langle 2 \rangle} \cdot \dfrac{1}{v1u}$ $\quad rad_i := (v1_i)^{\frac{1}{3}}$ $\quad Hu := z^{\langle 5 \rangle} \cdot \dfrac{1}{v4u} + r_0$ $\quad Hu_m := \max(Hu)$

$\max(rad) = 697 \cdot m$ **<== Max plume radius**

$Hu_m = 643 \cdot m$ **<== Plume Stabilization Height**

Note: Multiplication of v1, v2, v3, and v4 by v1u, v2u, v3u, and v4u is to reinstall units.

$v2 := z^{\langle 3 \rangle} \cdot \dfrac{1}{v2u}$ $\quad v3 := z^{\langle 4 \rangle} \cdot \dfrac{1}{v3u}$

Observed Cloud Rise Data:

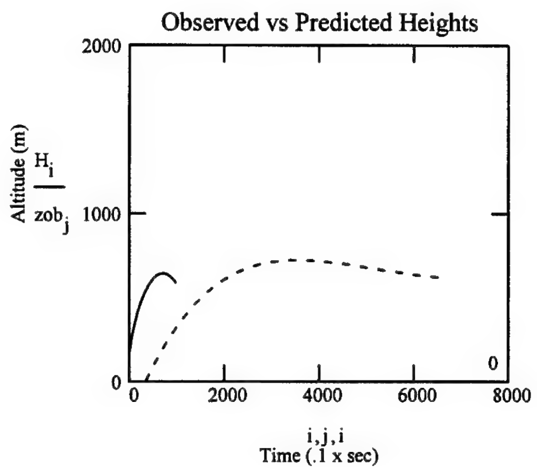
$t_{ob} := 11 \cdot 60$ $\quad j := 1 .. t_{ob} \cdot n + 1$ $\quad x_j := \dfrac{j}{60 \cdot n}$ $\quad (ft - it) \cdot n = 1 \cdot 10^3$ $\quad n = 10$

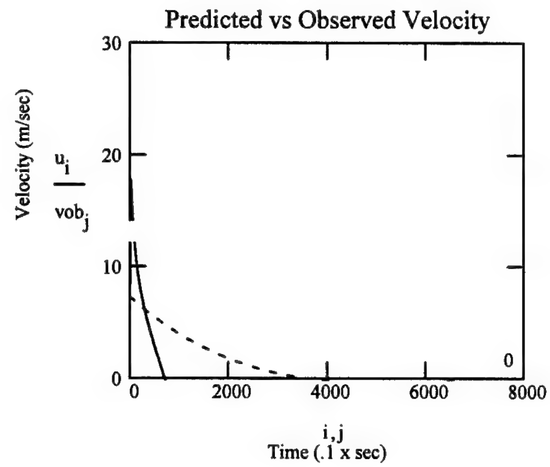
$zob_j := .0016 \cdot (x_j)^5 - .1495 \cdot (x_j)^4 + 4.9361 \cdot (x_j)^3 - 71.141 \cdot (x_j)^2 + 437.92 \cdot x_j - 229.17$ $\quad \max(zob) = 722.209$

$vob_j := \left[8.0 \cdot 10^{-3} \cdot (x_j)^4 - .598 \cdot (x_j)^3 + 14.8083 \cdot (x_j)^2 - 142.282 \cdot x_j + 437.92 \right] \cdot \dfrac{1}{60}$

$\rho_i := \rho_{00} - v3_i \cdot \dfrac{\rho_1}{g \cdot v1_i}$ $\quad u_i := \dfrac{v2_i}{\rho_i \cdot v1_i} \cdot sec \cdot m^{-1}$ $\quad \alpha = 1.118$

$H_i := Hu_i \cdot v4u$



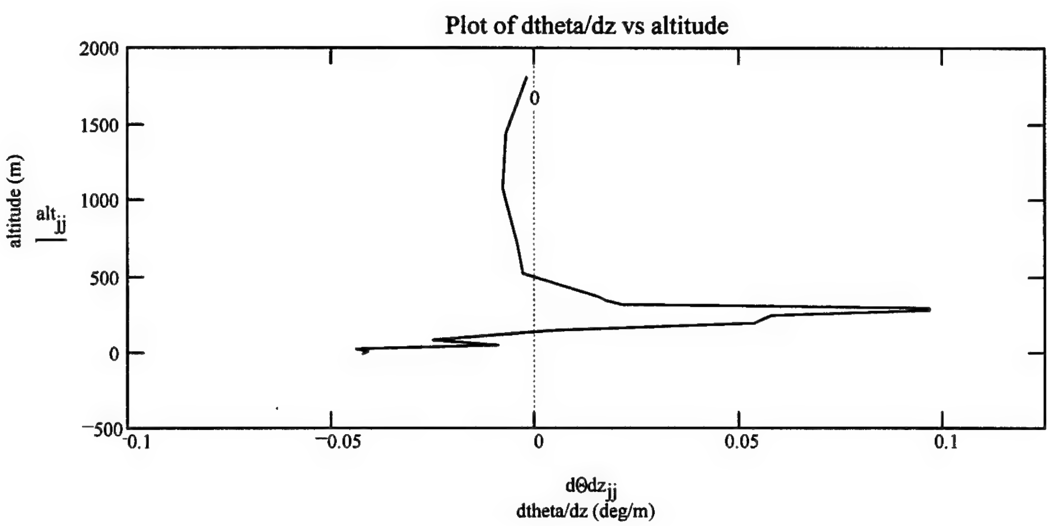


Divide x-axis values by $n = 10$ **to get time scale in seconds**

$jj := 1..18$

Determination of dtheta/dz:

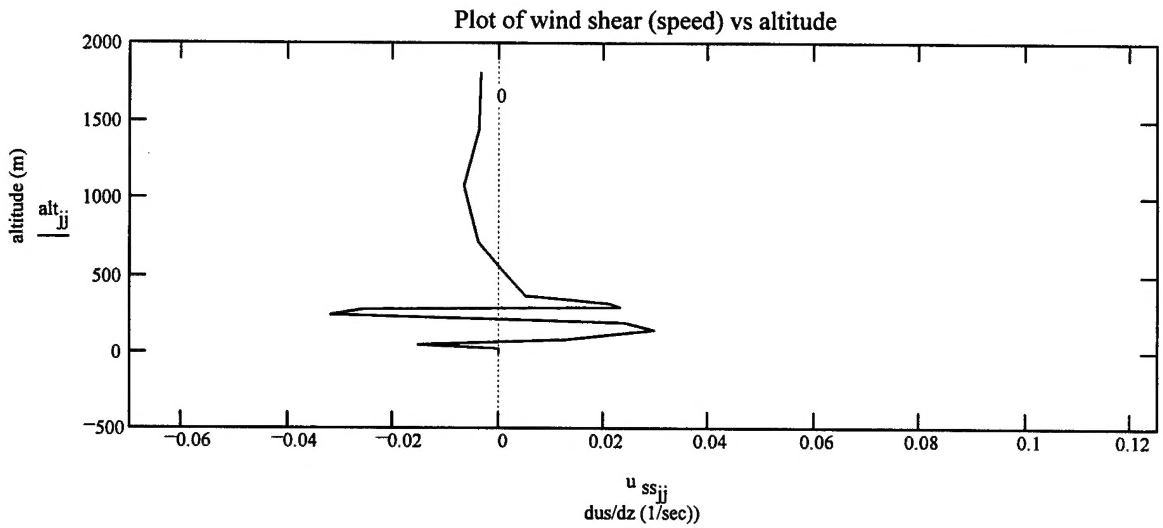
$$d\Theta dz_{jj} := \left(\frac{pt_{jj+1} - pt_{jj}}{alt_{jj+1} - alt_{jj}} \right) \cdot m \cdot K^{-1}$$

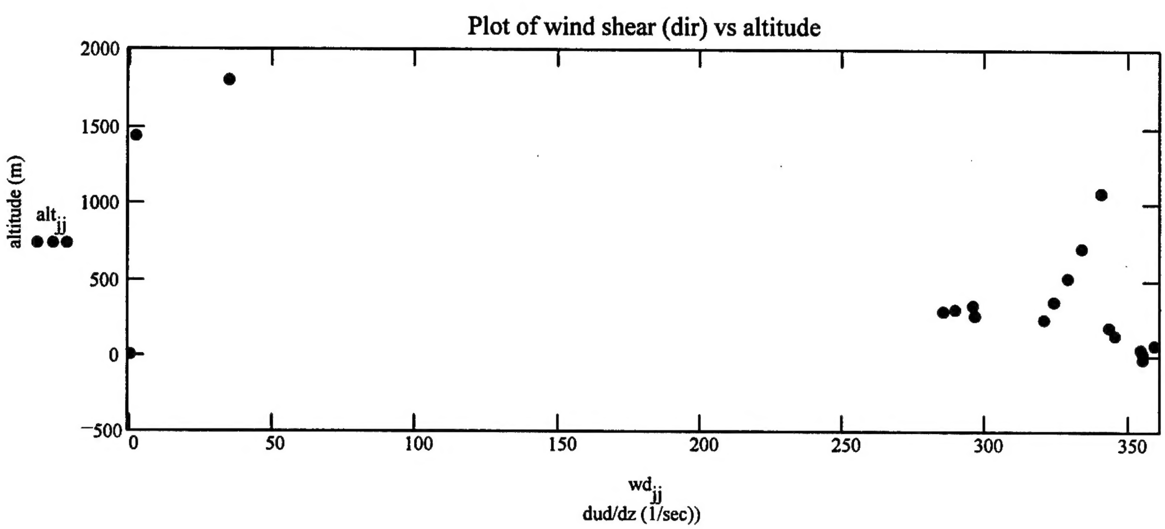


Determination of wind shear for speed and direction:

$$u_{ss_{jj}} := \frac{ws_{jj+1} - ws_{jj}}{alt_{jj+1} - alt_{jj}} \cdot sec$$

$$u_{sd_{jj}} := \frac{wd_{jj+1} - wd_{jj}}{alt_{jj+1} - alt_{jj}} \cdot m \cdot deg^{-1}$$



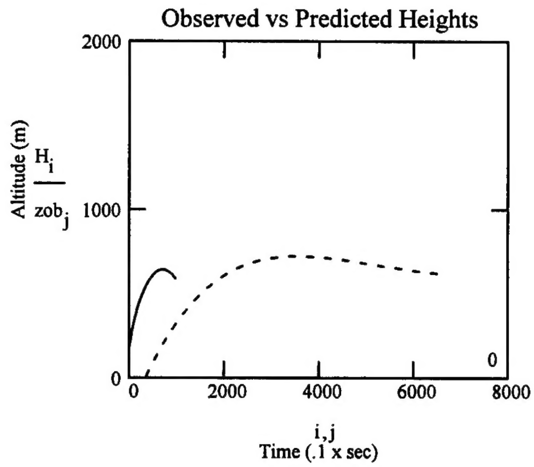


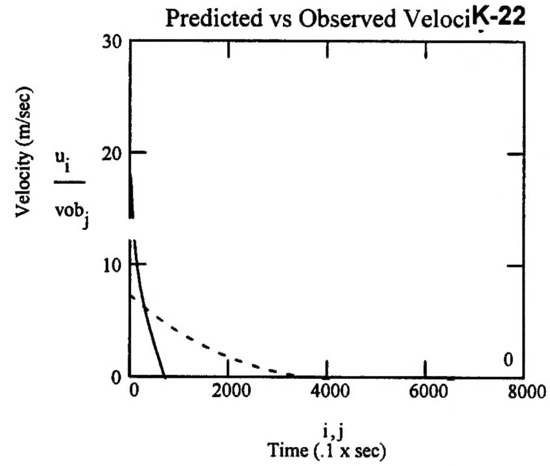
$ptj_{jj} := pt_{jj}$ $wsj_{jj} := ws_{jj}$ $wdj_{jj} := wd_{jj}$

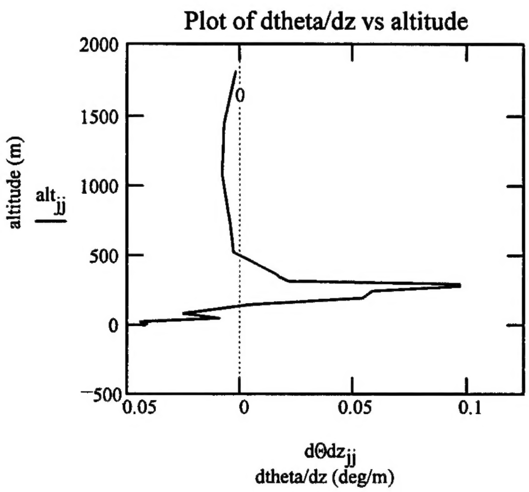
mean_pt := mean(ptj) var_pt := var(ptj)

mean_ws := mean(wsj) var_ws := var(wsj)

mean_wd := mean(wdj) var_wd := var(wdj)







mean_pt = 293.639•K

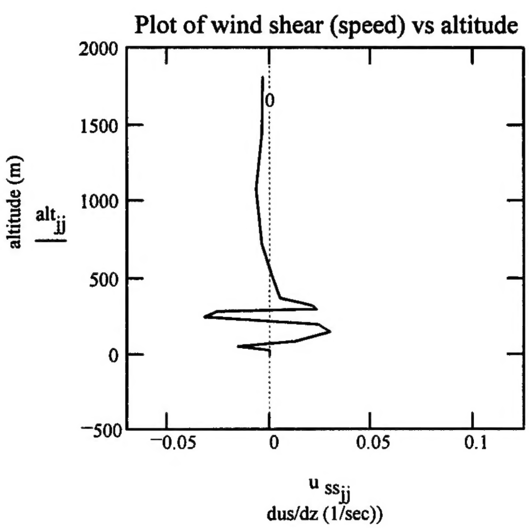
var_pt = 13.309•K^2

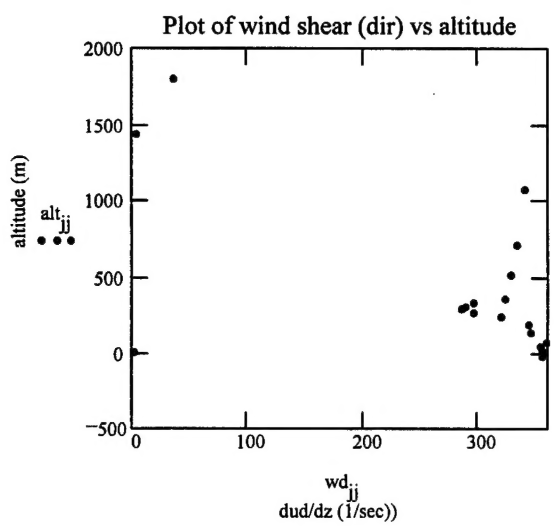
mean_ws = 4.389•m•sec^{-1}

var_ws = 2.209•m^2•sec^{-2}

mean_wd = 275.461

var_wd = 1.444•10^4





VITA

Captain Joseph D. Brands was born in Portland, OR on 27 March 1970. He graduated from Tigard High School in 1988. He attended and graduated from Oregon State University, earning a B.S. degree in Mechanical Engineering in 1993. He was commissioned a 2nd Lieutenant in the United States Air Force through the Air Force Reserve Officer Training Corps the same year.

He entered active duty in September of 1993, reporting to the Facilities Directorate at Arnold AFB in Tennessee. His duties included environmental engineering and project manager. While there, he had the opportunity to work on many pollution prevention initiatives including the base recycling program and hazardous waste reduction projects.

In may of 1996, he reported to the Air Force Institute of Technology at Wright-Patterson AFB to earn a M.S. degree in Engineering and Environmental Management. His area of study was air quality management with a supporting program of environmental sciences. Upon completion of his thesis he graduated in December of 1997.

His assignment following the Master's degree program is to Edwards AFB in California, where he will be the Chief of Maintenance Engineering for the 95th Civil Engineering Squadron.

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13. ABSTRACT *(Maximum 200 words)*
Space launches at Cape Canaveral Air Station (CCAS) and Vandeberg Air Force Base (VAFB) produce ground exhaust clouds from the solid rocket boosters and liquid hypergolic fuels containing several toxic substances. In order to estimate the health effects that would be imposed upon the public by scheduled launches, range safety officials rely on the Rocket Exhaust Effluent Diffusion Model (REEDM) to predict ground level concentrations of these substances. A drawback to the REEDM is its underprediction of the initial ground clouds stabilization height. This underprediction causes an overprediction of the ground level toxic substance concentrations. This thesis focused on increasing the accuracy of the clouds stabilization height. Therefore, a model was developed incorporating conservation principles of volume, momentum, and buoyancy to predict stabilization height values. As part of the model a predictive function for the coefficient of entrainment was developed based on meteorological conditions. the results of the model predicted the stabilization heights of ground clouds more accurately than the currently used REEDM. The mean square difference between predicted and observed stabilization heights for five CCAS and two VAFB launches were determined and show a significant improvement of the model from the REEDM predictions.

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